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Lateral Strength and Stiffness of Post and Pier Foundations

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Abstract. *The latest version of ASABE's shallow post and pier foundation design standard (ANSI/ASAE EP486.2) contains new procedures for determining the ultimate bearing, uplift and lateral strength capacities of the foundations. Presented in this paper are the theories and assumptions inherent in development of equations and methods for determining lateral movement and the ultimate lateral strength capacity of embedded piers and posts. Also presented are design examples and recommendations for adjustments to the recently completed standard.*

Keywords. *foundation, post foundation, pier, post-frame, pole frame, posts, soil strength.*

Introduction

In October 2012, ASABE released ANSI/ASAE EP486.2 *Shallow Post and Pier Foundation Design*. Initial work on this version of the EP began in 2005, and was undertaken to address several shortcomings of the previous version (ANSI/ASAE EP486.1) which was released in 1999. Shortcoming of the 1999 version included:

- Use of allowable soil stress limits of uncertain origin, and thus application of stress limits without certainty of the ultimate soil strength on which they were based or the factor of safety implicit in their use.
- The assumption that lateral soil stiffness (i.e., coefficient of horizontal subgrade reaction) increases linearly with depth regardless of soil type/layering.
- The assumption that at-grade forces (axial load, shear force, and bending moment) in a post/pier foundation are not dependent on below-grade deformations of the post/pier.
- A lack of adjustments to soil stress limits to account for building end use (risk category), or method used to identify soil type/properties.
- A lack of design equations and procedures for load and resistance factor design (LRFD).
- The assumption that the below grade portion of post/piers is infinitely rigid.
- The limitation that the groundline shear force V_G and groundline bending moment M_G must independently move the top of the foundation in the same direction (see figure 1).
- The requirement that the soil must be homogeneous with depth.
- Failure to account for water table location on foundation design.

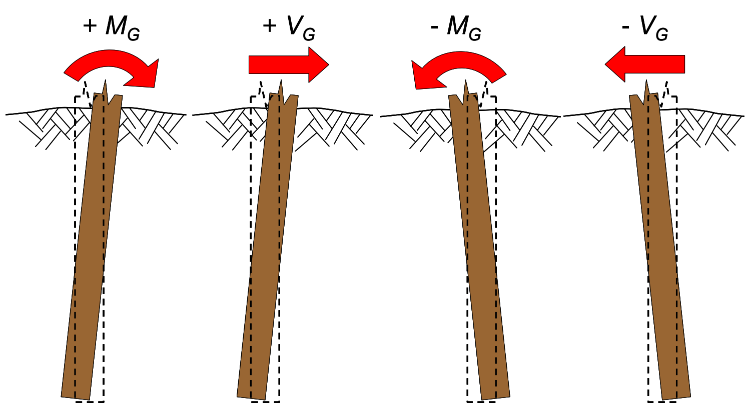


Figure 1. Groundline shear forces and groundline bending moments are given the same sign if they independently move the top of a foundation in the same direction.

In addition to addressing all of the previously listed shortcomings, the October 2012 version of ANSI/ASAE EP486 employs ultimate strength methodologies in determining the capacity of a shallow post/pier foundation under uplift, bearing and lateral loadings. This approach lends itself well to LRFD, and is a departure from previous editions of the EP.

Overviewed in this paper are procedures for calculating pier/post lateral stiffness and strength that were developed by the author for incorporation into ANSI/ASAE EP486.2. Specifically, this paper covers "simplified" and "universal" methods developed by the author for (a) estimating the groundline rotation and horizontal displacement of a post/pier foundation, and (b) determining the maximum groundline bending moment M_G and maximum groundline shear V_G that can simultaneously be applied to a post/pier foundation.

Simplified Versus Universal Methods

When it comes to determining the lateral stiffness and strength of a shallow post or pier foundation, two different approaches were developed for ANSI/ASAE EP486. The first approach utilizes a set of equations and is referred to as the ***Simplified Method*** as it does not require any special computer software, just your basic calculator. In many respects, this approach could be referred to as the traditional method because it mirrors past procedures.

The second approach relies on modeling soil with a series of simple springs. This approach requires structural analysis software and is referred to as the ***Universal Method***. Modeling soil behavior with simple springs is a *discrete* approach to analysis that has been utilized for well over a century. A summary of foundation-soil interaction models developed by researchers who have used this discrete approach has been provided by Maheshwari (2011). Within the post-frame building community, McGuire (1998) used a spring model to study the behavior of non-constrained posts subjected to ground-line shear forces and ground-line bending moments applied such that they caused below-grade post rotation in opposite directions (see figure 1). McGuire conducted his investigation to illustrate that when shear and bending moments are so applied, most equations used to calculate allowable embedment depth are not applicable.

The modifier "universal" was given to the soil spring method because the method can be used without restriction. Conversely, use of the Simplified Method equations for calculating both foundation stiffness and strength assumes the following:

1. Soil is homogeneous for the entire embedment depth.
2. Soil stiffness (i.e., the modulus of horizontal subgrade reaction, k) is either constant for all depths below grade or linearly increases with depth below grade.
3. Width of the below-grade portion of the foundation is constant. This generally means that there are no attached collars or footings that are effective in resisting lateral soil forces.

In addition, Simplified Method equations for calculating foundation stiffness assume that the below-grade portion of the pier/post has an infinite flexural rigidity (EI). This limits applicability of the equations to situations where soil stiffness is assumed to increase linearly with depth and:

$$d \leq 2\{EI/(2\,A_E)\}^{0.20} \tag{1}$$

or, where soil stiffness is assumed constant with depth and:

$$d \leq 2\{EI/(2\,E_{SE})\}^{0.25} \tag{2}$$

where:

d = depth of embedment
EI = flexural rigidity of the post/pier foundation
E_{SE} = effective Young's modulus of the soil
A_E = linear increase in effective Young's modulus with depth below grade

Equations 1 and 2 are based on work by Broms (1964a, 1964b).

Soil Properties

Young's Modulus for Soil, E_S.

Young's modulus, E_S, is used to predict the lateral movement of a foundation. E_S can be determined from laboratory tests or from in-situ soil tests. In general, it is best to reserve laboratory tests for backfills; that is, highly disturbed materials without a stress history. E_S for non-backfill materials is generally best estimated using field (in-situ) tests because of the significance of stress history on E_S and the difficulty of obtaining undisturbed soil samples for laboratory testing.

Common laboratory tests include triaxial compression tests conducted in accordance with ASTM D2166 and D2850. E_s for most cohesive soils can also be determined using an unconfined compression test in accordance with ASTM D3080.

In-situ tests for E_S include prebored pressuremeter tests (PBPMT), cone penetration tests (CPT), and standard penetration tests (SPT). Information and equations for determining Young's modulus from these field tests were compiled by the author and have been incorporated into ANSI/ASAE EP486.2.

In the absence of soil test data, the presumptive soil properties in Table 1 can be used. The presumptive values in Table 1 assume that E_S is constant with depth for silts and clays, and increases linearly with depth for sands and gravels. To calculate E_S for sands and gravels, multiple the A_E value in the second last column of Table 1 by depth, z. In equation form:

$$E_{S,z} = A_E\, z \tag{3}$$

where:

$E_{S,z}$ = E_S that is equal to zero at grade and increases linearly with depth z below grade, kPa, (lbf/in^2)

A_E =increase in Young's modulus per unit increase in depth z below grade, kN/m^3 (lbf/in^3)
z = depth below grade, in (m)

Table 1. Presumptive Soil Properties for Post and Pier Foundation Design from ANSI/ASAE EP486.2

Soil Type	Unified Soil Classification	Consistency	Moist unit weight, γ	Drained soil friction angle[(a)], ϕ'	Undrained soil shear strength[(b)], S_U	Young's modulus for soil[(c)(d)], E_S	Increase in Young's modulus per unit depth below grade[(c)(d)(e)], A_E		Poisson's ratio[(f)], ν
			lbf/ft^3	deg	lbf/in^2	lbf/in^2	$\frac{\text{lbf}}{\text{in}^2\text{-ft}}$	$\frac{\text{lbf}}{\text{in}^3}$	
Homogeneous inorganic clay, sandy or silty clay	CL	Soft	125	NA	3.5	3920	-	-	0.5
		Medium to Stiff	130		7	6160	-	-	
		Very Stiff to Hard	135		14	8400	-	-	
Homogeneous inorganic clay of high plasticity	CH	Soft	110	NA	3.5	1680	-	-	0.5
		Medium to stiff	115		7	2800	-	-	
		Very Stiff to Hard	120		14	4480	-	-	
Inorganic silt, sandy or clayey silt, varved silt-clay-fine sand of low plasticity	ML	Soft	120	NA	3.5	3920	-	-	0.5
		Medium to stiff			7	6160	-	-	
		Very Stiff to Hard			14	8400	-	-	
Inorganic silt, sandy or clayey silt, varved silt-clay-fine sand of high plasticity	MH	Soft	105	NA	3.5	1680	-	-	0.5
		Medium to stiff			7	2800	-	-	
		Very Stiff to Hard			14	4480	-	-	
Silty or clayey fine to coarse sand	SM, SC, SP-SM, SP-SC, SW-SM SW-SC	Loose	105	30	NA	-	440	37	0.3
		Medium to Dense	110	35		-	660	55	
		Very Dense	115	40		-	880	73	
Clean sand with little gravel	SW, SP	Loose	115	30	NA	-	880	73	0.3
		Medium to Dense	120	35		-	1320	110	
		Very Dense	125	40		-	1760	147	
Gravel, gravel-sand mixture, boulder-gravel mixtures	GW, GP	Loose	135	35	NA	-	2640	220	0.3
		Medium to Dense		40		-	3520	293	
		Very Dense		45		-	4400	367	
Well-graded mixture of fine- and coarse-grained soil: glacial till, hardpan, boulder clay	GW-GC GC, SC	Loose	120	35	NA	-	1320	110	0.3
		Medium to Dense	125	40		-	1760	147	
		Very Dense	130	45		-	2200	183	

(a) Rapid undrained loading will typically be the critical design scenario in these soils. Laboratory testing is recommended to assess clay friction angle for drained loading analysis.
(b) Loading assumed slow enough that sandy soils behave in a drained manner.
(c) Estimate of stiffness at a rotation of 1^{o} for use in approximating structural load distribution. Use values that are 1/3 of the tabulated values for serviceability limit state evaluations.
(d) Constant values of stiffness used for calculation of clay response. Stiffness increasing with depth from a value of zero used for calculation of sand response.
(e) Assumes soil is located below the water table. Double the tabulated A_E value for soils located above the water table.
(f) Poisson ratio of 0.5 (no volume change) assumes rapid undrained loading conditions.

Modulus of Horizontal Subgrade Reaction, *k*.

The modulus of horizontal subgrade reaction k is the ratio of average contact pressure (between foundation and soil) and the horizontal movement of the foundation. Modulus of subgrade reaction at depth z is equated to 2 times the effective Young's modulus for the soil at depth z divided by width of the foundation at depth z.

$$k = 2\, E_S / b \tag{4}$$

where:

k = modulus of horizontal subgrade reaction at depth z, kN/m^3 (lbf/in.^{3})
E_S = Young's modulus for soil at depth z, kPa (lbf/in.^{2})
= E_{SE} (effective Young's modulus at depth z) when backfill and unexcavated soil have different properties
b = face width of the foundation component (post/pier, footing or collar) at depth z, m (in.)
z = depth below the ground surface, m (in.)

Equation 4 is based on elastic theory and recommended by Pyke and Beikae (1984). It is similar in form to the standard equation for the modulus of *vertical* subgrade reaction k_V, which from elastic theory is given as:

$$k_V = q / S_i = E_s / [C_S\, b\, (1 - v^2)] \quad (5)$$

where: q is the equivalent uniform load on the footing, S_i is the immediate settlement of a point on the footing surface, E_S is Young's modulus for the soil, C_S is a combined footing shape and rigidity factor, b is the characteristic width of the footing, and v is Poisson's ratio. C_S is equated to 0.79 for rigid circular footings and to 0.82 for rigid square footings. For rigid rectangular footings with length/width ratios of 2, 5 and 10, C_S is equal to 1.12, 1.6 and 2.0, respectively (NFEC, 1986, Table 1 page 7.1-212).

Although Pyke and Beikae (1984) found the modulus of horizontal subgrade reaction to be equal to 2.3, 2.0, and 1.8 times E_S / b for Poisson's ratios of zero, 0.33, and 0.5, respectively, they recommend equating k to 2.0 E_S / b for all Poisson ratio values for practical purposes. Pyke and Beikae point out that this equation neglects friction between the foundation and soil, and also neglects the decrease in pressure on the back side of the foundation as it undergoes lateral movement. They note that a value of the order of 2.0 E_S / b is not unreasonable as it is about twice the value obtained by considering a strip footing acting on the surface of a half space.

Effective Young's Modulus, E_{SE}

Equation 4 requires use of an effective Young's modulus when a foundation is surrounded by backfill material with properties that differ from those of the surrounding, unexcavated soil (figure 2).

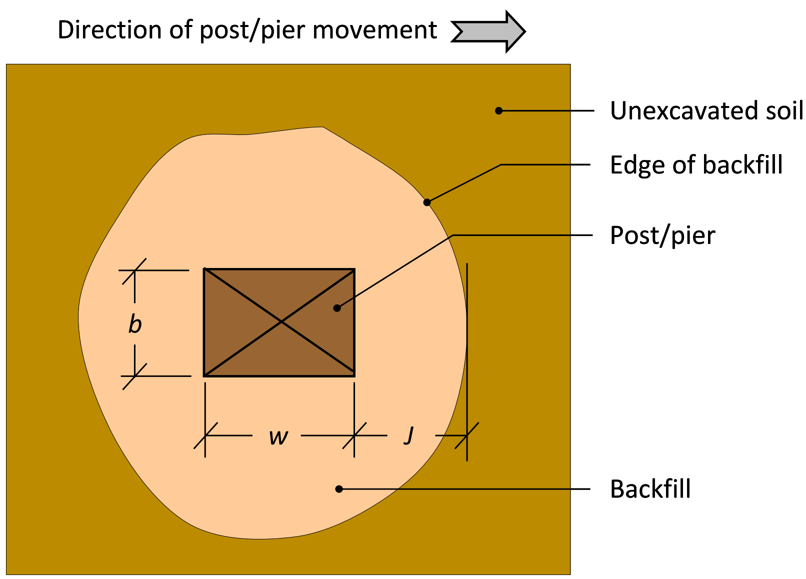


Figure 2. Top view of foundation showing distance *J* between the foundation component (i.e., post, pier, footing, or collar) and the edge of the backfill.

Effective Young's modulus, E_{SE}, can be calculated as:

$$E_{SE} = \frac{1}{I_S / E_{S,B} + (1-I_S)/ E_{S,U}} \quad \text{for } 0 < J < 3b \qquad (6a)$$

$$E_{SE} = E_{S,B} \quad \text{for } J \geq 3b \qquad (6b)$$

$$E_{SE} = E_{S,U} \quad \text{for } J = 0 \qquad (6c)$$

where:

$E_{S,B}$ = E_S for backfill at depth z
$E_{S,U}$ = E_S for the unexcavated soil surrounding the backfill at depth z
I_S = strain influence factor, I_S, dimensionless

= $[\ln(1 + J/b)]/1.386$ for $0 < J < 3b$ (7)

J = distance (measured in the direction of laterally foundation movement) between the edge of the backfill and the face of the foundation component at depth z (see figure 2)
b = width of the post, collar, footing that is surrounded by the backfill at depth z

The strain influence factor is the fraction of total lateral displacement that is due to soil straining within a distance J of the face of the foundation.

When the foundation is surrounded by unexcavated soil, $J = 0$ and $E_{SE} = E_{S,U} = E_S$ of the unexcavated soil (equation 6c). Such is the case when a post is driven into the soil, or a helical pier is turned into the soil.

Equation 6c also applies to those portions of a foundation that are entirely backfilled with concrete or controlled low-strength material (CLSM). In this case, E_{SE}, is equated to E_S for the soil surrounding the concrete or CLSM.

Derivation of equation 6a is provided in Appendix A. An example E_{SE} calculation is provided in Appendix B.

Ultimate Lateral Soil Resistance, p_U

The maximum horizontal pressure that can be applied to the soil by the face a pier/post is defined as the ultimate lateral soil resistance, p_U, and can be calculated as:

$$p_{U,z} = 3\sigma'_{v,z} K_P + (2 + z/b)\, c\, K_P^{\,0.5} \quad \text{for } 0 \leq z < 4b \qquad (8)$$

$$p_{U,z} = 3\,(\sigma'_{v,z} K_P + 2\, c\, K_P^{\,0.5}) \quad \text{for } z \geq 4b \qquad (9)$$

where:

$p_{U,z}$ = ultimate lateral resistance p_U at depth z
K_P = coefficient of passive earth pressure, dimensionless
= $(1 + \sin\phi)/(1 - \sin\phi)$
ϕ = soil friction angle, degrees
c = soil cohesion at depth z
b = face width of foundation at the groundline
$\sigma'_{v,z}$ = effective vertical stress at depth z
= $\sigma_{v,z} - u_z = \gamma z - u_z$
$\sigma_{v,z}$ = total vertical stress at depth z
= γz
γ = moist unit weight of soil
u_z = pore water pressure at depth z
= γ_w •(distance the water table is above depth z)
γ_w = water unit weight = 62.4 lbf/ft^3 = 0.0361 lbf/in^3

Equations 8 and 9 equate ultimate lateral soil resisting pressure p_U to three times the Rankine passive pressure. Although basing resisting pressure solely on passive pressure would appear to neglect the active earth-pressure acting on the back of the foundation and side friction, the factor of three by which the passive pressure is increased is based on observed ultimate loads – ultimate loads which were most likely influenced by forces acting on all sides of the foundation system.

Passive pressure due to soil cohesion is assumed to increase from 1/3 its full value at the ground surface to its full value at a depth of 4 b. This partially accounts for the reduced soil containment at the soil surface and less than full mobilization of the soil due to the likelihood of foundation-soil detachment near the surface. The quantity $2cK_P^{0.5}$ in equations 8 and 9 is the Rankine passive pressure due to soil cohesion.

For cohesionless soils, equations 8 and 9 both reduce to:

$$p_{U,z} = 3\ \sigma'_{v,z}\ K_P \tag{10}$$

For cohesive soils, equations 8 and 9 can be approximated as:

$$p_{U,z} = 3\ S_U\ (1 + z/(2b)) \qquad \text{for } 0 \leq z < 4b \tag{11}$$

$$p_{U,z} = 9\ S_U \qquad \text{for } z \geq 4b \tag{12}$$

where: S_U is undrained soil shear strength at depth z. S_U is numerically equal to cohesion, c, for a saturated clay soil. The value of 9 S_U is approximately equal to three times $2S_UK_P^{0.5}$ when ϕ is equal to 32 degrees. Information and equations for determining S_U from laboratory and field tests were compiled by the author and have been incorporated into ANSI/ASAE EP486.2. Presumptive values for S_U are provided in Table 1.

Modeling with Soil Springs

As previously noted, the universal method of analysis involves the use of soil springs to model soil behavior. Outlined in this section are details for spring placement and properties.

Soil Spring Location

To locate soil springs, first draw horizontal sectioning lines wherever there is an abrupt change in soil type, backfill type, and/or width of the post/pier foundation. Each layer resulting from this sectioning must be modeled with at least one soil spring. It is recommended that soil spring spacing t should not exceed 2w where w is the side width of a rectangular post/pier (see figure 3) or diameter of a round post/pier.

Locating soil springs is illustrated in Figures 3 and 4. Figure 3 shows six soil springs being used to model a non-constrained post in a multi-layered soil. Since the footing is not attached to the post, there is no need for a soil spring to model soil in contact with the footing. It is important to note that such an assumption ignores friction between the post and footing. Although three springs are being used to model the resistance provided by each soil layer, two per layer would be sufficient given that the thickness of each soil layer does not appear to exceed more than 4 times the width w.

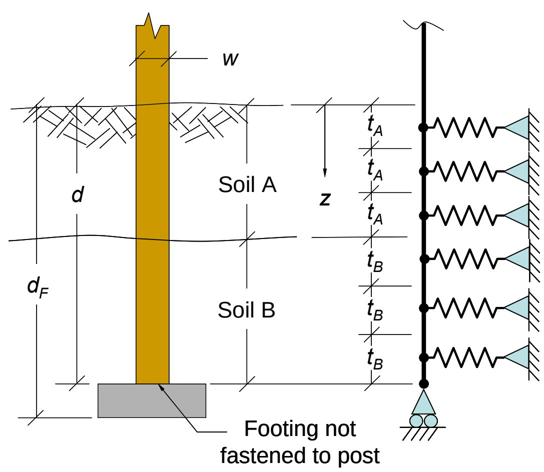


Figure 3. Modeling a non-constrained post in a layered soil. Footing not attached to rest of foundation.

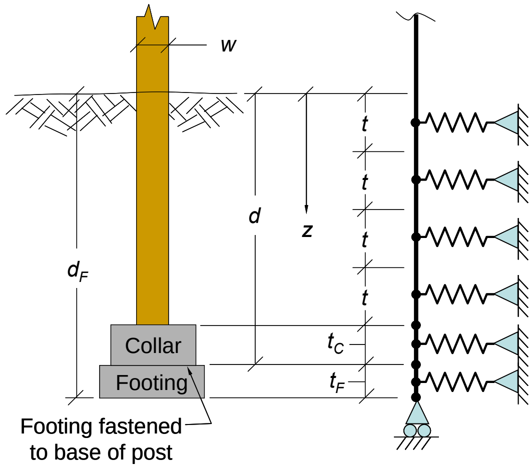
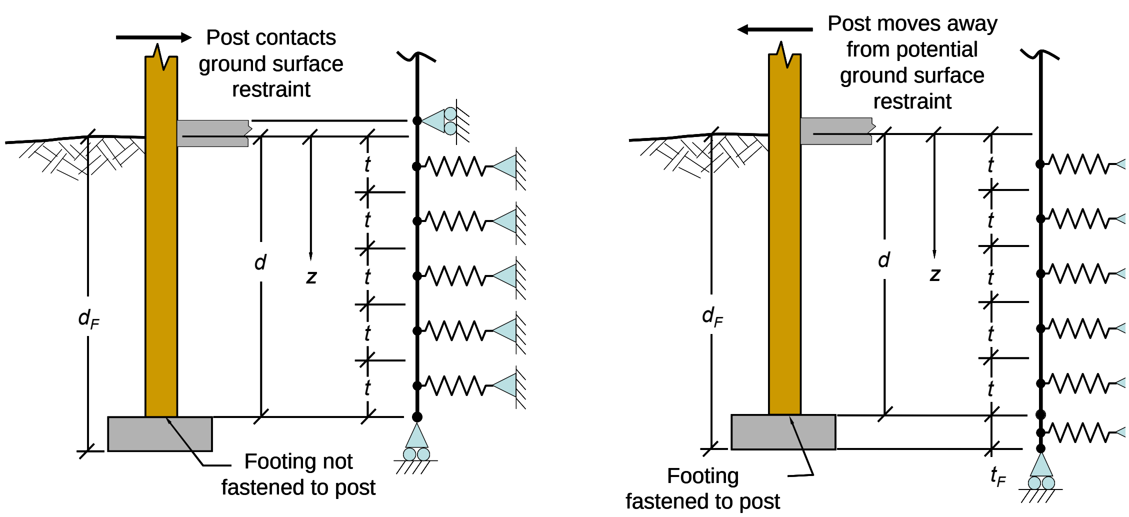


Figure 4. Modeling soil behavior when both footing and collar are attached to the post

Figure 4 shows the modeling of a non-constrained post that has an attached footing and an attached collar. Note that individual springs are required for both the footing and the collar because they each have different widths relative to the post.

In addition soil springs, other restraints associated with post/pier foundation modeling include placement of a horizontal roller support at the foundation base as shown in Figures 3 and 4. Such a support ignores friction between the foundation and underlying soil.

Resistance provided by surface restraint(s) must also be modeled. Figure 5 shows an embedded post that abuts a slab-on-grade. To model the restraint that the slab provides when the post moves toward the slab, the slab is modeled as a vertical roller support (figure 5(a)). Because the slab only abuts the inside of the post and is not attached to the post, it does not apply a force to the post when the post moved away from the slab, and thus is simply modeled as a non-constrained post (figure 5(b)).



(a) **(b)**

Figure 5. Modeling an embedded post abutting a slab-on-grade when the post moves (a) toward the slab, and (b) away from the slab

Soil Spring Properties

All soil springs are assumed to exhibit linear-elastic behavior until a point of soil failure is reached, at which point the force in the soil spring stays constant as the spring undergoes additional deformation. A graphical depiction of this behavior is shown in 6.

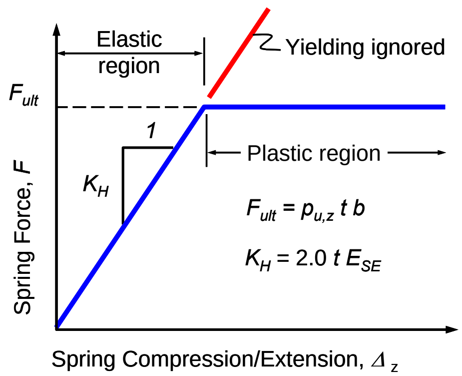


Figure 6. Load-displacement relationship for a soil spring.

The initial stiffness, K_H, of an individual soil spring is given as:

$$K_H = k\ t\ b = 2.0\ t\ E_{SE} \tag{13}$$

and the ultimate strength, F_{ult}, of an individual soil spring is given as:

$$F_{ult} = p_{u,z}\ t\ b \tag{14}$$

where:

- K_H = stiffness of a horizontal spring used to model the resistance to lateral post/pier movement provided by a soil layer with thickness t in contact with a foundation element of width b, kN/m (lbf/in.)
- k = modulus of horizontal subgrade reaction at spring location, kN/m^3 (lbf/in.^{3})
- t = thickness of the soil layer represented by the spring, m (in.)
- b = width of foundation at spring location, m (in.)
- E_{SE} = effective Young's modulus for soil at spring location, kN/m^2 (lbf/in^2)
- F_{ult} = ultimate strength of an individual soil spring representing a soil-foundation contact area with a width b and height t, kN (lbf)
- $p_{u,z}$ = ultimate lateral resistance p_U at spring location, kN/m^2 (lbf/in.^{2})

For structural analyses used to determine load distribution within a building frame, yielding of soil springs is ignored. In other words, the stiffness of a soil spring is assumed to equal K_H regardless of the load applied to the spring. Thus, the value of F_{ult} displayed in figure 6 is not needed during the structural analysis phase of the design process.

Ignoring soil spring yielding during structural analyses is consistent with the modeling of steel frame members and all other components that do not exhibit linear-elastic behavior at high loads. The sole purpose of a structural analysis is to determine load distribution under service load conditions. When properly sized, no component (soil, steel, or otherwise) should be loaded to levels near those associated with plastic behavior or failure.

Soil failure planes associated with the ultimate lateral capacity of the foundation are almost entirely located in the unexcavated soil surrounding the backfill. For this reason. Use unexcavated soil properties in calculations of $p_{u,z}$

An example structural analysis of a foundation utilizing soil springs is provided in Appendix C.

Equations for Approximating Foundation Displacements and Soil Pressures

The lateral displacement of the below-grade portion of a post/pier foundation and associated soil pressures induced by a groundline bending moment M_G and groundline shear V_G can be estimated using equations in this section. Use of these equations has been referred to as the simplified method. As previously noted, during equation derivation it was assumed that: (1) the below-grade portion of the foundation has an infinite flexural rigidity (EI), (2) effective Young's modulus for the soil, E_{SE}, is either constant for all depths below grade or linearly increases with depth below grade, and (3) the width of the below-grade portion of the foundation is constant. The latter generally means that there are no attached collars, uplift anchors or footings that are effective in resisting lateral soil forces.

Variables appearing in this section are defined as follows:

M_G = bending moment applied to foundation at grade (a.k.a. groundline bending moment)
V_G = shear force applied to foundation at grade (a.k.a. groundline shear force)
Δ = horizontal displacement of foundation at grade
= 0 (zero) for surface-constrained foundation
θ = rotation of the infinitely stiff foundation
d_R = depth from the ground surface to the pivot point for foundation rotation
= depth below the ground surface at which foundation does not displace horizontally
= 0 (zero) for surface-constrained foundation
E_{SE} = effective Young's modulus of the soil surrounding the foundation
A_E = linear increase in effective Young's modulus with depth below grade
p_z = contact pressure between soil and foundation at a depth z

Non-constrained posts/piers with linearly increasing E_{SE}

Equations 15 through 18 assume that the post/pier foundation is not restrained at grade, and that E_{SE} increases linearly with soil depth, and is numerically equal to $A_E\, z$ (figure 7).

$$d_R = \frac{d(3V_G\, d + 4M_G)}{4V_G\, d + 6M_G} \tag{15}$$

$$\theta = \frac{12V_G\, d + 18M_G}{d^4 A_E} \tag{16}$$

$$\Delta = \frac{9V_G\, d + 12M_G}{d^3\, A_E} \tag{17}$$

$$p_z = 6z(6M_G\, z/d + 4V_G\, z - 3d\, V_G - 4M_G)/(d^3 b) \tag{18}$$

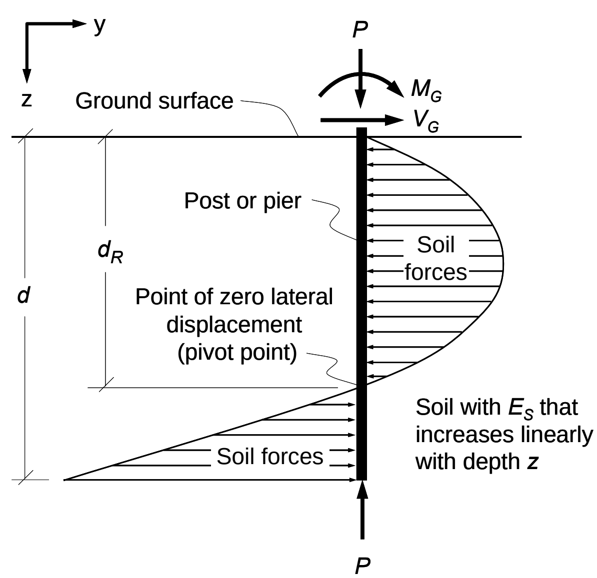


Figure 7. Forces acting on a non-constrained post/pier of fixed width *b* when E_{SE} increases linearly with depth.

Constrained posts/piers with linearly increasing E_{SE}

Equations 19 and 20 assume that the post/pier is restrained at the groundline and that E_{SE} increases linearly with soil depth, and is numerically equal to $A_E\, z$ (figure 8).

$$\theta = \frac{2M_G}{d^4 A_E} \tag{19}$$

$$p_z = 4z^2 M_G / (d^4 b) \tag{20}$$

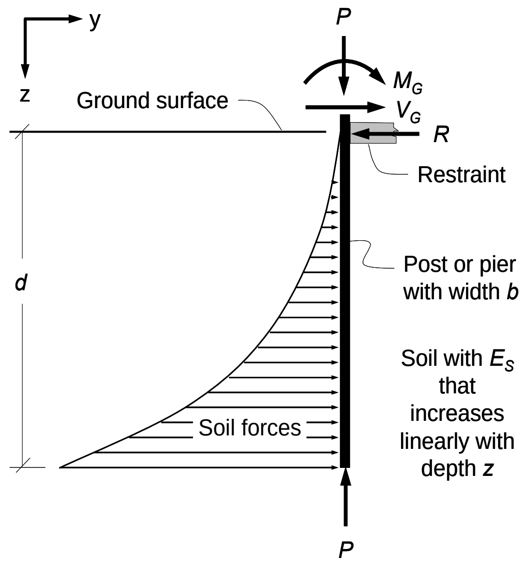


Figure 8. Forces acting on a ground surface-constrained post/pier of fixed width *b* when E_{SE} increases linearly with depth.

Non-constrained posts/piers with constant E_{SE}

Equations 21 through 24 assume that the post/pier foundation is non-constrained and that E_{SE} remains constant with depth (figure 9).

$$d_R = \frac{d(2V_G\, d + 3M_G)}{3V_G\, d + 6M_G} \tag{21}$$

$$\theta = \frac{3V_G\, d + 6M_G}{d^3 E_{SE}} \tag{22}$$

$$\Delta = \frac{2V_G\, d + 3M_G}{d^2 E_{SE}} \tag{23}$$

$$p_z = (12M_G\, z/d + 6V_G\, z - 4dV_G - 6M_G)/(d^2 b) \tag{24}$$

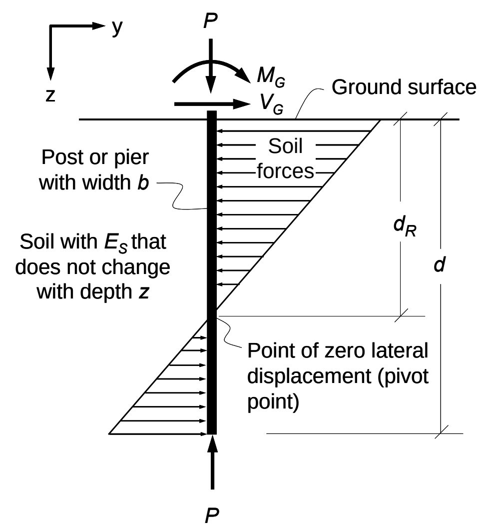


Figure 9. Forces acting on a non-constrained post/pier of fixed width b when ESE is constant with depth.

Constrained posts/piers with constant E_{SE} (figure 10)

Equations 25 and 26 assume that the post/pier foundation is constrained at grade and E_{SE} remains constant with depth (figure 10).

$$\theta = \frac{1.5M_G}{d^3 E_{SE}} \qquad (25)$$

$$p_z = 3zM_G/(d^3 b) \qquad (26)$$

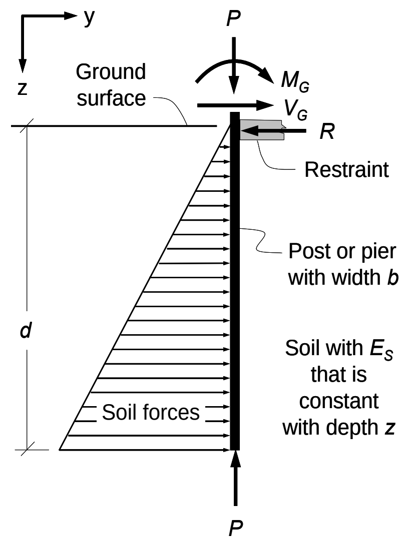


Figure 10. Forces acting on a ground surface-constrained post/pier of fixed width b when ESE is constant with depth.

Lateral Strength Overview

V_U – M_U Failure Envelope

As the groundline shear force V_G and groundline bending moment M_G applied to the top of a post (or pier) foundation are increased, the pressure applied to the foundation by the soil increases. This increase in soil pressure at a particular depth will continue until the ultimate lateral soil resisting pressure p_U at that depth (see equations 8 through 12) is reached. Once this point is reached, there is no further increase in pressure applied to the foundation by the soil at that depth.

The ultimate strength of a post or pier foundation is reached when *all* soil in contact with the foundation has reached its ultimate lateral soil resisting pressure. The groundline shear force V_G and groundline bending moment M_G when this state is reached are respectively defined as the ultimate groundline shear capacity V_U and the ultimate groundline moment capacity M_U of the foundation as limited by soil strength.

For any foundation, the ultimate groundline moment capacity, M_U, is dependent on the groundline shear force induced in the foundation. Thus there is (in theory) an infinite number of V_U - M_U combinations for each non-constrained foundation design. These combinations can be represented with a V_U -M_U *envelope* on a plot of groundline shear V_G versus groundline bending moment M_G (figure 11).

Plotted in figure 11 is a V_U -M_U envelope for a foundation with a 4.5 inch width and 48 inch depth. Additionally, the foundation is surrounded by a cohesionless soil with a moist unit weight of 110 lbm/ft^3 and soil friction angle of 35 degrees. Any combination of groundline shear V_G and groundline bending moment M_G that falls within the V_U -M_U envelope will not exceed the ultimate capacity of the foundation.

Maintaining a proper sign convention is important. Groundline shear forces and groundline bending moments are given the same sign when they independently rotate the foundation in the same direction (figure 1).

The two shaded regions in figure 11 identify loadings in which groundline bending moment and groundline shear have the same sign. Although these regions comprise a relatively small area of the V_U - M_U envelope, the vast majority of loadings on non-constrained foundations are located in these regions.

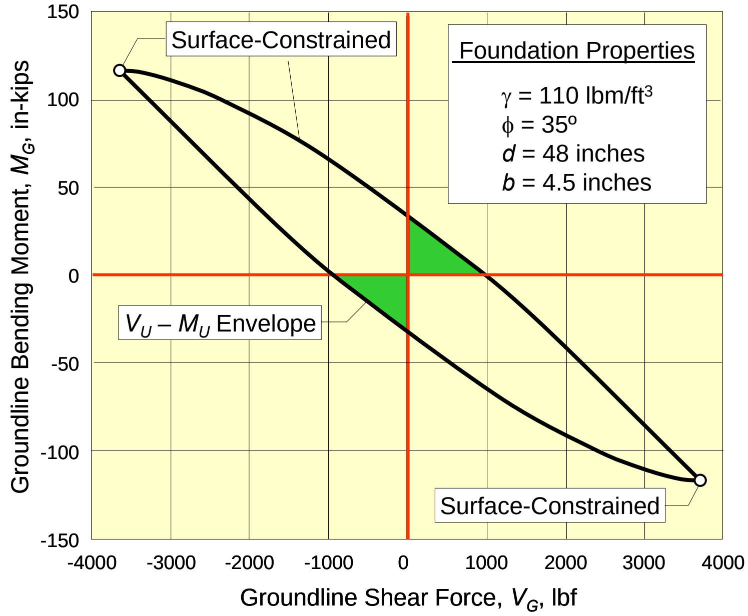


Figure 11. V_U - M_U envelope for a post/pier foundation.

The two most extreme points on the $V_U – M_U$ envelope represent the $V_U – M_U$ combinations associated with the restraint of the foundation at or just above the ground surface (i.e., constrained or surface-constrained foundation). In this case, the groundline shear force is the force in the foundation at a point just below the surface restraint, and is opposite in sign to the groundline bending moment.

Governing Equations

As previously stated, any combination of groundline shear V_G and groundline bending moment M_G that falls within the V_U-M_U envelope will, in theory, not exceed the ultimate capacity of the foundation.

In practice, V_G and M_G are limited by governing design equations. For allowable stress design (ASD) groundline shear V_G and groundline bending moment M_G are identified as V_{ASD} and M_{ASD}, respectively, and are limited as follows:

$$f_L\, V_{ASD} \leq V_U \tag{27}$$

$$f_L\, M_{ASD} \leq M_U \tag{28}$$

For load and resistance factor design (LRFD) groundline shear V_G and groundline bending moment M_G are identified as V_{LRFD} and M_{LRFD}, respectively, and are limited as follows:

$$V_{LRFD} \leq V_U\, R_L \tag{29}$$

$$M_{LRFD} \leq M_U\, R_L \tag{30}$$

Where:

M_U = Ultimate groundline moment capacity for the foundation (as limited by soil strength)
V_U = Ultimate groundline shear capacity for the foundation (as limited by soil strength)
f_L = ASD factor of safety for lateral strength assessment from Table 2
R_L = LRFD resistance factor for lateral strength assessment from Table 2
M_{ASD} = M_G due to an ASD load combination
V_{ASD} = V_G due to an ASD load combination
M_{LRFD} = M_G due to a LRFD load combination
V_{LRFD} = V_G due to a LRFD load combination
M_G = groundline bending moment (bending moment in foundation at the ground surface)
V_G = groundline shear force (shear force in foundation at the ground surface)

Table 2. LRFD Resistance Factors and ASD Safety Factors for Lateral Strength Assessment (a)

Soil	Method used to determine ultimate lateral soil resistance, $p_{U,z}$	LRFD resistance factor for lateral strength assessment, R_L	ASD safety factor for lateral strength assessment, f_L
Cohesionless (SP, SW, GP. GW, GW-GC, GC, SC, SM, SP-SM, SP-SC, SW-SM, SW-SC)	Equation 10 with soil friction angle ϕ determined from laboratory direct shear or axial compression tests	$0.86 - 0.01 \cdot \phi$	$1.4/(0.86 - 0.01 \cdot \phi)$
	Equation 10 with soil friction angle ϕ determined from SPT data	$0.66 - 0.01 \cdot \phi$	$1.4/(0.66 - 0.01 \cdot \phi)$
	Equation 10 with soil friction angle ϕ determined from CPT data	$0.76 - 0.01 \cdot \phi$	$1.4/(0.76 - 0.01 \cdot \phi)$
	Equation 10 with presumptive soil friction angle ϕ from Table 1	$0.61 - 0.01 \cdot \phi$	$1.4/(0.61 - 0.01 \cdot \phi)$
	Equation 10 with presumptive soil friction angle ϕ from Table 1, with soil type verified by construction testing	$0.82 - 0.01 \cdot \phi$	$1.4/(0.82 - 0.01 \cdot \phi)$
	Pressuremeter test (PMT)	0.56	2.5
Cohesive (CL,CH, ML, MH)	Equations 11 and 12 with undrained shear strength S_U determined from laboratory compression tests	0.68	2.1
	Equations 11 and 12 with undrained shear strength S_U determined from PBPMT data	0.68	2.1
	Equations 11 and 12 with undrained shear strength S_U determined from CPT data	0.68	2.1
	Equations 11 and 12 with undrained shear strength S_U determined from in-situ vane tests	0.68	2.1
	Equations 11 and 12 with presumptive undrained shear strength S_U from Table 1	0.44	3.2
	Equations 11 and 12 with presumptive undrained shear strength S_U from Table 1 with soil type verified by construction testing	0.68	2.1
	Pressuremeter test (PMT)	0.68	2.1

(a) For buildings and other structures that represent a low risk to human life in the event of a failure (e.g., ANSI/ASCE-7 Category I structures), resistance factors may be increased 25 percent (multiplied by 1.25), and corresponding safety factors may be reduced 20 percent (multiplied by 0.80). In all cases, the adjusted resistance factor is limited to a maximum value of 0.93 and the adjusted safety factor is limited to a minimum value of 1.50.

Resistance and safety factors for lateral strength capacity in Table 2 are based on work by Foye, et al., (2006a, 2006b) and on similar factors compiled in the AASHTO LRFD Bridge Design Specifications. The factors are a function of the method used to arrive at values for ultimate lateral soil resistance, $p_{U,z}$. Simply put, the more accurate the method, the lower the factor of safety.

Lateral capacities in cohesionless soils increase exponentially with friction angle, and thus small variances in estimated friction angle have an amplified effect on these capacities as friction angle increases (Foye, et al., 2006a). For this reason, a smaller (more conservative) resistance factor is required for greater friction angles.

ANSI/ASAE EP486.2 contains two sets of resistance and safety factors for lateral strength assessment, one for the Simplified Method of analysis and one for the Universal Method of analysis. The thought was that assumptions inherent in the development of "Simplified Method" equations dictated more conservative resistance and safety factors than those associated with the Universal Method of analysis. In reality, the assumptions inherent in the derivation of the "Simplified Method" equations will typically result in lower estimates of M_U and V_U and thus the resistance and safety factors should, if anything, be less conservative then those associated with the universal method of analysis. For this reason, only one set of resistance and safety factors are given in Table 2.

M_U and V_U via the Simplified Method

The equations in this Section are only applicable to foundations that have a fixed face width and are surrounded by soil that is homogeneous for the entire embedment depth.

Equations for non-constrained foundations were set up with ultimate bending moment M_U as the dependent variable and ultimate shear force V_U as an independent variable. This procedure begins with the calculation of d_{RU} which is the distance from the ground surface to the point of foundation rotation (i.e., the point at which there is no lateral foundation translation below grade) when failure is imminent. The value of d_{RU} is a function of total foundation depth d, soil properties required to determine p_U, and groundline shear V_G. Groundline shear is used to establish a minimum required ultimate shear strength V_U. If d_{RU} is greater than d, the depth of embedment is insufficient to resist the groundline shear V_G (i.e., there is not enough soil pushing on the foundation to counter the groundline shear). Once d_{RU} has been calculated and found to be less than d, M_U can be determined. If the calculation for M_U returns a negative number, the embedment depth d must be increased. A negative sign implies that the only way to handle the shear force V_U with the existing depth is to apply a moment that rotates the foundation in the opposite direction of the applied shear force.

To establish a V_U - M_U envelope line for a non-constrained foundation, simply calculate M_U for more than one V_U value as shown in figure 12.

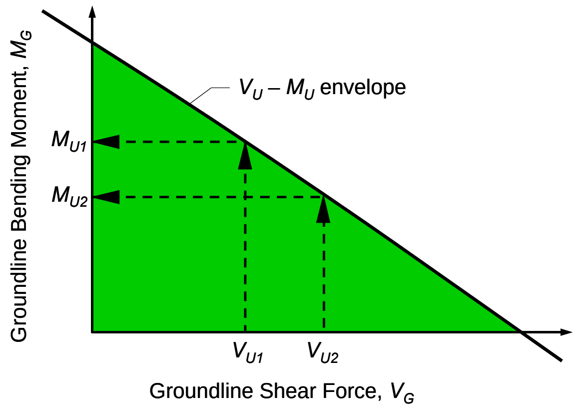


Figure 12. Equations for the Simplified Method for non-constrained foundations require selection of a V_U value to determine M_U.

Variables used in this section have been previously defined with the exception of the following:

d_{RU} = Depth from ground surface to the ultimate pivot point (i.e., the point below grade at which the foundation does not move horizontally under ultimate load).

S_{LU} = Increase in the ultimate lateral force per unit depth applied to a foundation by a cohesionless soil

Non-Constrained Foundation in Cohesionless Soils

The ultimate moment M_U that can be applied at the groundline to a post/pier foundation that is not constrained at the groundline and is embedded in cohesionless soil (figure 13) is:

$$M_U = S_{LU}\,(d^3 - 2\,d_{RU}^3)\,/\,3 \quad (31)$$

where:

$d_{RU} = (V_U / S_{LU} + d^2/2)^{0.5} \le d$
$S_{LU} = 3\,b\,K_P\,\gamma$
$K_P = (1 + \sin\phi)/(1 - \sin\phi)$
$V_U = V_{LRFD} / R_L$ for LRFD
$V_U = f_L\,V_{ASD}$ for ASD

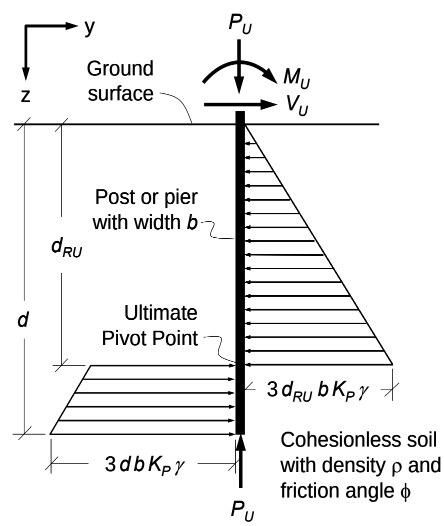


Figure 13. Forces acting on a non-constrained foundation of fixed width *b* in cohesionless soil at failure.

If shear force V_U is zero and there is a nonzero bending moment acting on the foundation, the foundation will rotate at a point below the surface equal to 0.707 d when Rankine soil pressures for cohesionless soils are acting. As V_U is increased, the point of rotation will lower (i.e., the ratio of d_{RU} to d will increase).

If shear V_G and moment M_G rotate the top of the foundation in opposite directions, a negative value must be input for V_{LRFD} (or V_{ASD}). This will move the point of rotation closer to the surface and d_{RU} will be less than 0.707 d.

Non-Constrained Foundation in Cohesive Soils

The ultimate moment M_U that can be applied at the groundline to a post/pier foundation that is not constrained at the groundline and is embedded in cohesive soil (figure 14) is:

$$M_U = b\, S_U\, (4.5\, d^{\,2} - 6\, d_{RU}^{\,2} - d_{RU}^{\,3}/(2b)) \tag{31}$$

where:

$d_{RU} = [64\, b^2 + 4V_U/(3S_U) + 12\, b\, d]^{1/2} - 8\, b \leq d$

and

$d_{RU} < 4b$

The preceding equations apply when d_{RU} is less than 4b and the force distribution shown in figure 14(a) applies. If d_{RU} from the preceding equation is greater than 4b (in which case the force distribution shown in figure 14(b) applies) then:

$$M_U = 9\, b\, S_U\, (d^{\,2}/2 - d_{RU}^{\,2} + 16\, b^{\,2}/9\,) \tag{32}$$

where:

$d_{RU} = V_U/(18\, b\, S_U) + d/2 + 2\, b/3 \leq d$

and

$d_{RU} \geq 4b$

In both cases:

$V_U = V_{LRFD}/R_L$ for LRFD

$V_U = f_L\, V_{ASD}$ for ASD

If shear V_{LRFD} (or V_{ASD}) and moment M_{LRFD} (or M_{ASD}) rotate the top of the foundation in opposite directions, input a negative value for V_{LRFD} (or V_{ASD}).

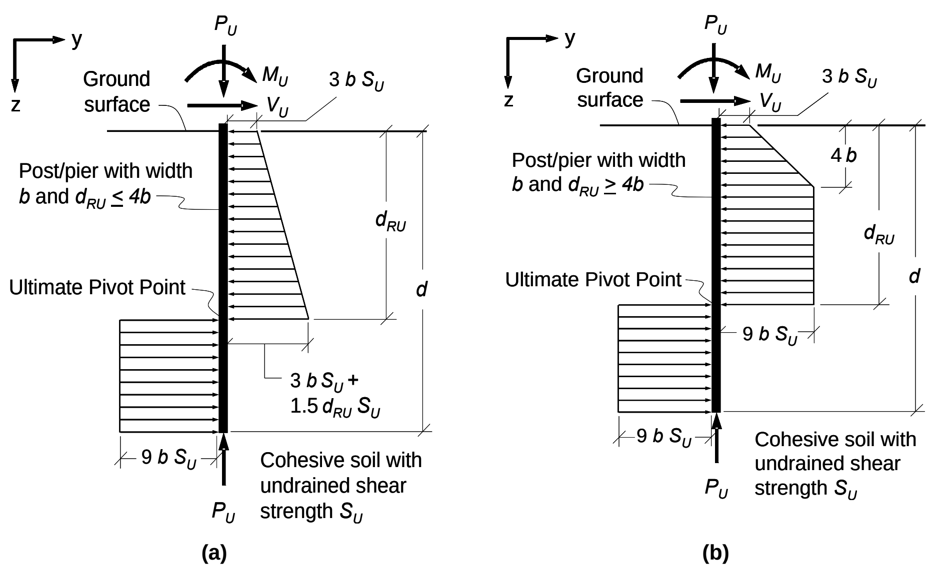


Figure 14. Forces acting on a non-constrained foundation of fixed width *b* in cohesive soil at failure (a) when d_{RU} is less than 4 *b*, and (b) when d_{RU} is greater than 4 *b*.

For calculation of the ultimate bending moment that can be applied to a non-constrained pier/post in cohesive soil, the force applied by the soil to the foundation per unit depth is assumed to equal 9 S_U b below the point of post/pier rotation. Above the point of rotation, a force of 3 S_U b is applied at the soil surface. This force increases at a rate of 1.5 S_U z. If 4b is less than d_{RU} the maximum applied soil force 9 S_U b will be reached above the point of post/pier rotation as shown in Figure 14(b). If 4b is greater than d_{RU} the soil force above the point of rotation reaches a maximum value at the point of rotation of S_U (3b + 1.5d_{RU}) as shown in Figure 14(a).

Non-Constrained Foundation in Any Soil

The ultimate moment M_U that can be applied at the groundline to a post/pier foundation that is constrained at the groundline and for which d_{RU} is greater than 4b (figure 15) is:

$$M_U = S_{LU}\,(d^{3} - 2\,d_{RU}^{3})\,/\,3 + 6\,b\,c\,K_P^{\,0.5}\,(d^{\,2}/\,2 - d_{RU}^{\,2} + b^{\,2}\,/\,4) \qquad (33)$$

where:

$d_{RU} = [A^2 + V_U\,/\,S_{LU} + \ dA + d^{\,2}/2 + A\ b/\,2]^{\,0.5} - A \leq d$
$d_{RU} > 4b$
$A = 2c\,/\,(K_P^{\,0.5}\ \gamma)$
$S_{LU} = 3\ b\ K_P\ \gamma$
$K_P = (1 + \sin\phi)/(1 - \sin\phi)$
$V_U = V_{LRFD}\,/\,R_L$ for LRFD
$V_U = f_L\ V_{ASD}$ for ASD

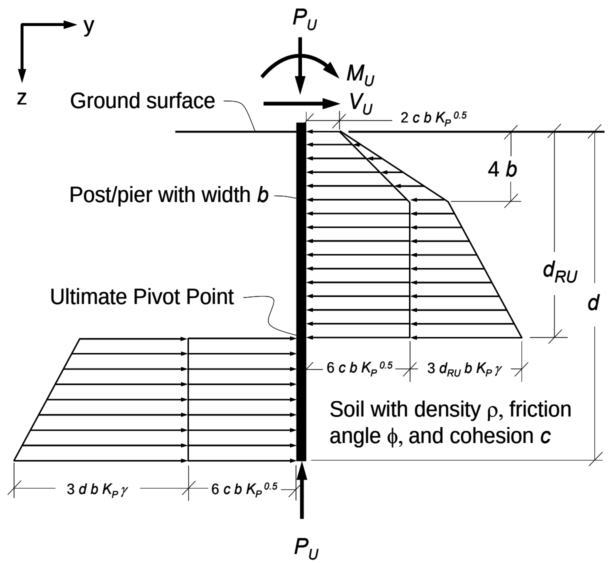


Figure 15. Forces acting on a non-constrained foundation of fixed width *b* in a homogenous soil at failure.

If shear V_{LRFD} (or V_{ASD}) and moment M_{LRFD} (or M_{ASD}) rotate the top of the foundation in opposite directions, input a negative value for V_{LRFD} (or V_{ASD}).

Equations for calculating the ultimate lateral load capacity of a pier/post in mixed soils requires tests to obtain both soil cohesion and friction angle under identical conditions (e.g. both drained). It is important that these conditions accurately reflect field conditions and do not overestimate soil strength as soil moisture content changes.

Constrained Foundation in Cohesionless Soils

The ultimate moment M_U that can be applied at the groundline to a post/pier foundation that is constrained at the groundline and is embedded in cohesionless soil (figure 16) is:

$$M_U = d^{\,3}\ b\ K_P\ \gamma \qquad (34)$$

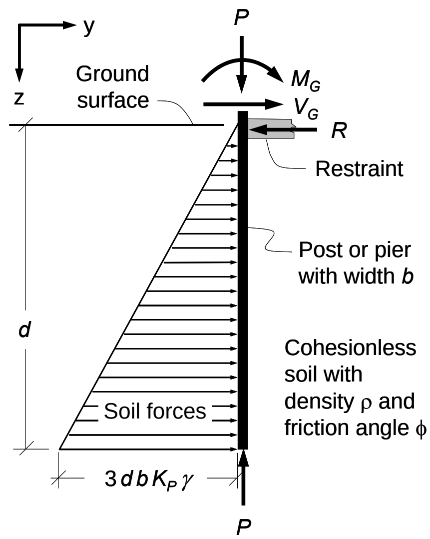


Figure 16. Forces acting on a constrained foundation of fixed width *b* in cohesionless soil at failure.

Constrained Foundation in Cohesive Soils

The ultimate moment M_U that can be applied at the groundline to a post/pier foundation that is constrained at the groundline and is embedded in cohesive soil (figure 17) is:

$$M_U = b\,S_U\,(4.5\,d^2 - 16\,b^2) \qquad \text{for } d \geq 4b \tag{35}$$

and

$$M_U = b\,d^2\,S_U\,(3/2 + d/(2b)) \qquad \text{for } d \leq 4b \tag{36}$$

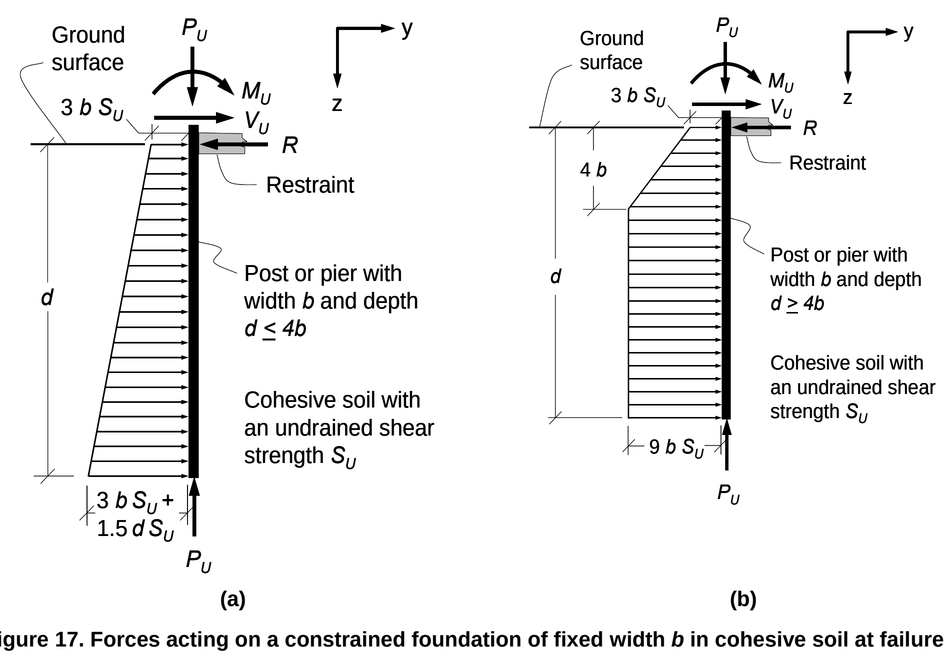


Figure 17. Forces acting on a constrained foundation of fixed width *b* in cohesive soil at failure (a) when *d* is less than 4 *b*, and (b) when *d* is greater than 4 *b*.

For calculation of the ultimate bending moment that can be applied to a constrained pier/post in cohesive soil, the force applied by the soil is assumed to equal 3 $S_U\, b$ at the soil surface and to increase at a rate of 1.5 $S_U\, z$ until a maximum of 9 $S_U\, b$ is reached at which point the force applied by the soil per unit depth remains at 9 $S_U\, b$. Where 4*b* exceeds *d*, the force acting on the foundation per unit depth will not reach 9 $S_U\, b$; instead it will reach a maximum at depth *d* of $S_U\,(3b + 1.5d)$.

Constrained Foundation in Any Soil

The ultimate moment M_U that can be applied at the groundline to a post/pier foundation that is constrained at the groundline and is embedded in any soil (figure 18) is:

$$M_U = d^3 b\, K_P\, \gamma + bcK_P^{\,0.5}\,(3d^2 - 32b^2/3) \qquad \text{for } d \geq 4b \qquad (37)$$

and

$$M_U = d^3\, b\, K_P\, \gamma + b\, d^2 c\, K_P^{\,0.5}\,(1 + d/(3b)) \qquad \text{for } d \leq 4b \qquad (38)$$

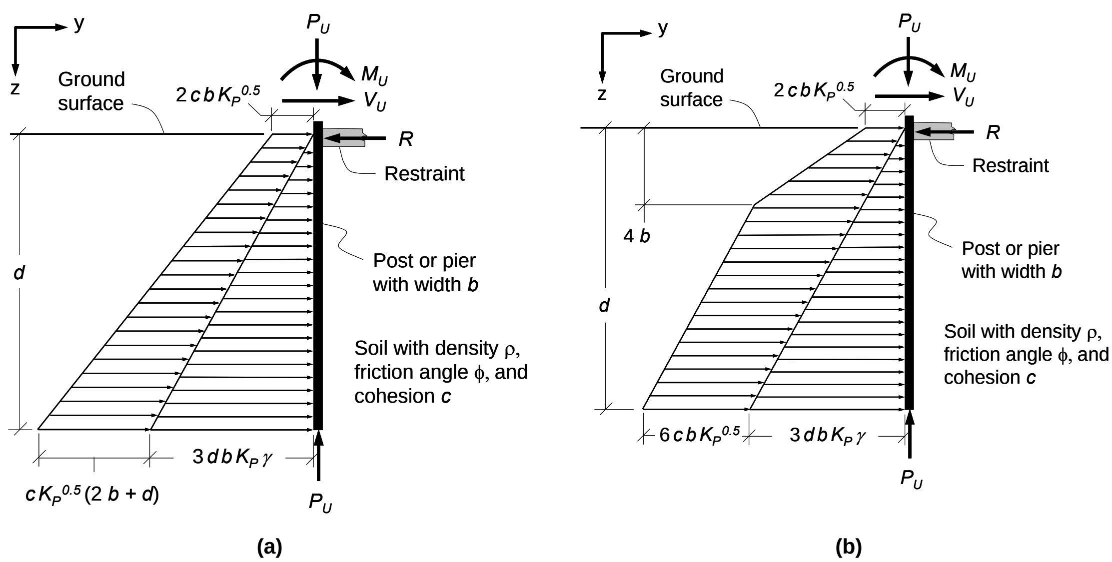


Figure 18. Forces acting on a constrained foundation of fixed width *b* in a homogenous soil at failure (a) when *d* is less than 4 *b*, and (b) when *d* is greater than 4 *b*.

Equations for calculating the ultimate lateral load capacity of a pier/post in mixed soils requires tests to obtain both soil cohesion and friction angle under identical conditions (e.g. both drained). It is important that these conditions accurately reflect field conditions and do not overestimate soil strength as soil moisture content changes.

Design Examples

Example analyses utilizing some of the equations in this section are provided in Appendices E and F.

M_U and V_U via the Universal Method

Conditions at Ultimate Lateral Capacity

Each soil spring is assumed to exhibit linear-elastic behavior until F_{ult} is reached, at which point the spring is assumed to undergo a plastic state of strain with the force in the soil spring remaining at F_{ult}. The lateral strength capacity of a foundation (as limited by soil strength) is reached when *all* springs acting on the foundation have reached their maximum ultimate strength capacity F_{ult}. In other words, a foundation has reached its lateral strength capacity when there is not a single remaining soil spring that can take additional load.

The groundline shear V_G and groundline bending moment M_G that will result in a plastic state of strain in all soil springs are defined respectively as the ultimate groundline shear capacity V_U and ultimate groundline moment capacity M_U for the foundation.

The key to determining M_U and V_U for any foundation is identifying on which side of the foundation each soil spring is pushing. At loads less than a foundation's ultimate capacity (i.e., prior to the yielding of all soil springs), the direction that many soil springs act is a function of the bending stiffness of the foundation relative to the stiffness of the surrounding soil, and some of these directions can switch as the applied loads increase as shown in figure 19.

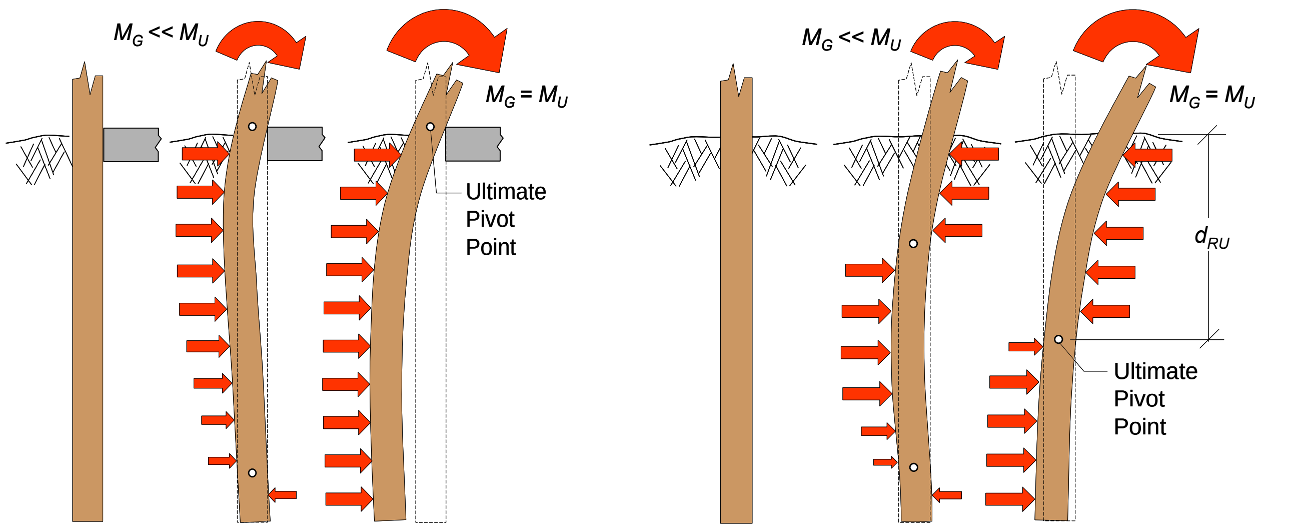


Figure 19. Surface-constrained (left) and non-constrained (right) post foundations subjected to a groundline bending moment. At ultimate lateral capacity ($M_G = M_U$) there is no more than one pivot point (i.e., the ultimate pivot point) located below grade.

Once all soil springs have yielded (i.e., once the foundation has reached its ultimate capacity), the foundation will pivot about a single point, herein referred to as the *ultimate pivot point* (figure 19). The ultimate pivot point is also identified as the *point of zero lateral displacement under ultimate load*. All soil springs located above an ultimate pivot point act in the same direction. Likewise, all springs located below an ultimate pivot point act in the same direction.

It’s important to note that the *ultimate* pivot point’s location is not a function of the foundation’s bending stiffness, nor is it a function of soil spring stiffness K_H (as previously stated, prior to reaching ultimate capacity, locations of zero lateral displacement in non-constrained posts are a function of the foundation’s bending stiffness and soil spring stiffness). This means that M_U and V_U for any foundation can be determined without knowledge of foundation bending properties or soil spring stiffness.

M_U and V_U for a Specified Ultimate Pivot Point Location

Each modeling spring represents a soil layer with thickness *t*. When the ultimate pivot point is located at the interface between two of these soil layers (see figure 20) or the ultimate pivot point is located above the soil surface or below the foundation, M_U and V_U can be calculated as:

$$M_U = \sum_{i=1}^{N} z_i F_{ult,i} \tag{39}$$

$$V_U = -\sum_{i=1}^{n} F_{ult,i} \tag{40}$$

where:

- M_U = Ultimate groundline moment capacity of the foundation (as limited by soil strength). Positive when acting clockwise.
- V_U = Ultimate shear capacity (as limited by soil strength) of the foundation at a point just below the foundation restraint. Positive when acting to the right.
- n = Number of springs used to model the soil surrounding the foundation.

$F_{ult,i}$ = Ultimate strength of spring *i*. Positive when pushing to the right.
z_i = Absolute distance between groundline and spring *i*.

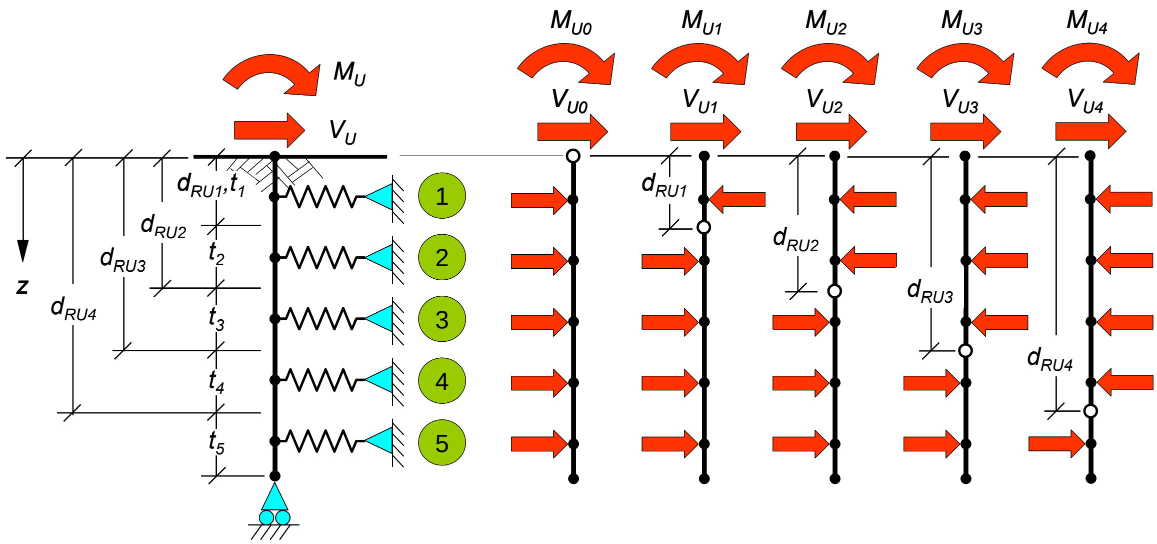


Figure 20. When ultimate pivot points are located at the interface between soil layers modeled with different soil springs, Equations 39 and 40 can be used to calculate M_U and V_U, respectively.

Equation 40 is obtained by summing forces in the horizontal direction on a free body diagram of the below-grade portion of a foundation. Equation 39 is obtained by summing moments about the groundline on the same free body diagram.

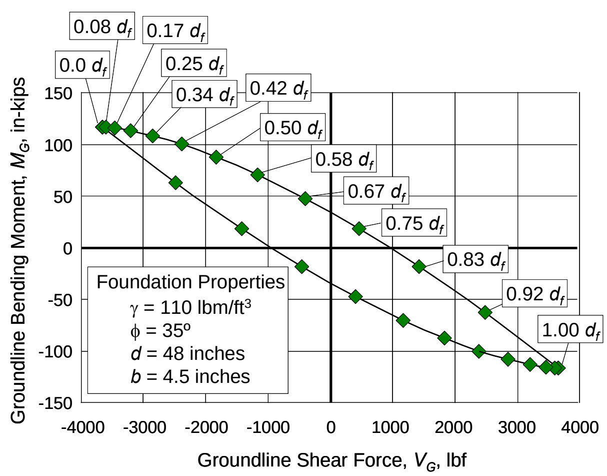


Figure 21. V_U - M_U envelope obtained by applying equations 39 and 40 for 13 different ultimate pivot point locations (12-spring model). "Boxed" values identify ultimate pivot point locations (d_{RU} values).

Figure 21 contains a V_U – M_U envelope obtained by applying equations 39 and 40 to all 13 ultimate pivot point locations associated with a 12 soil spring model (11 locations between springs plus locations at the groundline and foundation base). To obtain the full V_U – M_U envelope shown in figure 21, signs are switched on all thirteen "V_U , M_U" values. This is equivalent to switching the directions of all soil springs at each ultimate pivot point location.

For design purposes, the entire V_U – M_U envelope need not be constructed. Calculating M_U and V_U for three or so ultimate pivot points in the ½ d_f to $^7/_8$ d_f range, enables construction of a M_U -V_U envelope line that would cover most loadings associated with a non-constrained foundation. The deeper value of $^7/_8$ d_f is associated with foundations that have an attached footing, bottom collar, and/or some other mechanism that results in the base of the foundation having a much greater effective width than the rest of the foundation.

M_U and *V_U* for a Specified *M_G/V_G* Value

When M_G and V_G have the same sign, the foundation is adequate under lateral loads if $M_U \geq M_{ASD}\, f_L$ and $V_U \geq$

$V_{ASD}\ f_L$ for Allowable Stress Design, and $M_U \geq M_{LRFD}/R_L$ and $V_U \geq V_{LRFD}/R_L$ for Load and Resistance Factor Design.

Checking if these inequalities have been met is straight forward once a V_U -M_U envelope plot exists. For example, figure 22 shows the results of two different structural analyses involving the same foundation; one ASD and the other LRFD. A quick scan of this plot reveals that the foundation is adequate for the LRFD loading but not for the ASD loading.

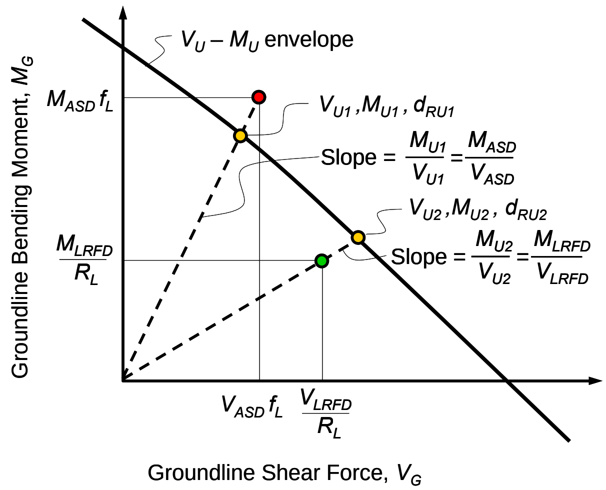


Figure 22. Using a V_U - M_U envelope to check the adequacy of a foundation under two different loadings.

In figure 22, "V_{U1}, M_{U1}" is the point on the V_U - M_U envelope that is numerically closest to coordinate point "$V_{ASD}\ f_L$, $M_{ASD}\ f_L$". "V_{U1}, M_{U1}" lies on a line drawn through the origin and "$V_{ASD}\ f_L$, $M_{ASD}\ f_L$". Stated differently, the closest "V_U, M_U" point to "$V_{ASD}\ f_L$, $M_{ASD} f_L$" is the one whose M_U/V_U value equals M_{ASD}/V_{ASD}. More generically, the closest "V_U, M_U" point to a particular "V_G, M_G" point is one whose M_U/V_U value equals M_G/V_G. Rearranging yields the equality:

$$M_U = V_U\ (M_G/V_G) \tag{41}$$

As will be demonstrated in the following paragraph, equation 41 makes it possible to determine if a foundation is adequate without having to first establish a V_U - M_U envelope plot like that shown in figure 21.

Figure 23a shows a nonconstrained post with M_G and V_G applied at the groundline. Figure 23b shows V_G located a distance M_G/V_G above the groundline. From a statics perspective, the diagrams in Figures 23a and 23b are equivalent. As force V_G in Figure 23b is increased, soil springs will begin to yield. As a spring yields, it is replaced with an equivalent force as shown in Figure 23c. Force V_G can be increased until all but one soil spring has reached its ultimate capacity F_{ult}. The value of V_G when this point is reached is defined as the ultimate groundline shear capacity of the foundation V_U (Figure 23d). Once V_U is established, M_U is calculated by direct application of equation 41.

The spring that has not reached its ultimate capacity F_{ult} (when V_U is reached) is the spring that represents the soil layer in which the ultimate pivot point is located. For this reason, the spring is referred to as the *pivot* spring. It follows that the pivot spring is simultaneously representing soil forces applied to both sides of the foundation as shown in Figure 23e. Because these forces (1) counteract each other, and (2) individually cannot exceed F_{ult}, the pivot spring itself will always have a load less than F_{ult}. The only time this would not be the case is when the ultimate pivot point is located exactly at the interface between soil layers represented by different springs (figure 20).

Given that the forces in all soil springs that have yielded are known, the only unknowns in Figure 23d are V_U and the force in the pivot spring. Thus, V_U can be calculated by summing moments about the point at which the pivot spring attaches to the foundation, and the force in the pivot spring can be determined by summing moments about the point at which V_U is applied (i.e., at a distance M_G/V_G from the groundline).

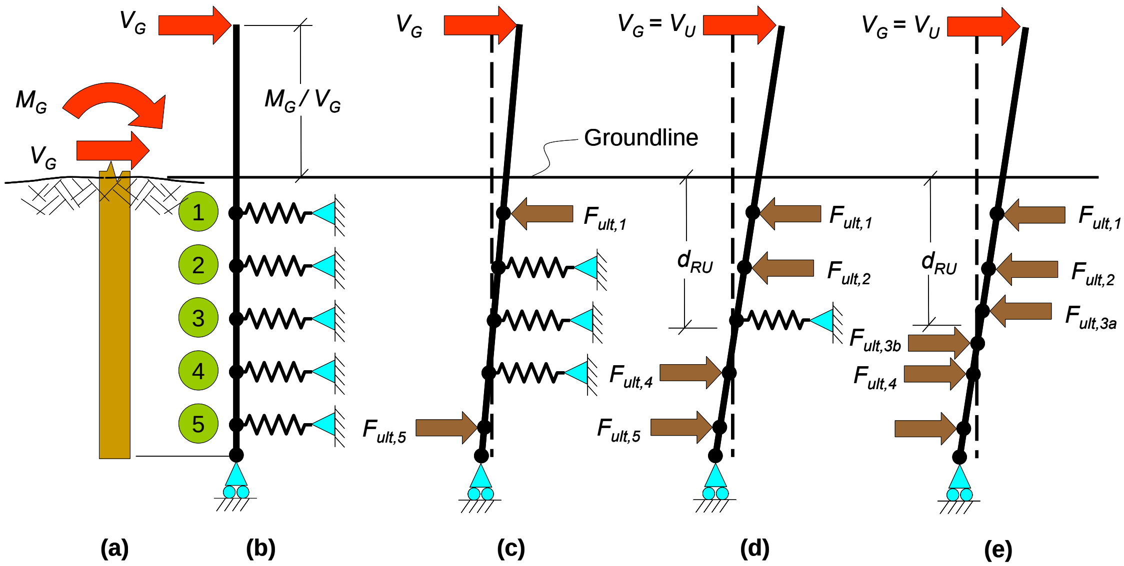


Figure 23. (a) Groundline shear V_G and groundline bending moment M_G, (b) equivalent load applied to spring model of foundation, (c) soil springs yield under increased load, (d) ultimate capacity of foundation is reached when all but one soil spring reaches its ultimate strength, (e) spring that doesn't reach its ultimate load is replaced by two opposing forces that represent force applied by soil yielding on both sides of the foundation.

It is evident that the procedure for determining V_U (and thus M_U) is very straightforward if one knows which one of the soil springs is the pivot spring. In practice, this can be determined by trial and error. If the wrong spring is selected, the absolute value of the force calculated for that spring will exceed the spring's F_{ult} value.

Example Determination of M_U and V_U for a Specified M_G/V_G Value

A nonconstrained post foundation with a uniform width of 4.5 inches, depth of 48 inches, located in cohesionless soil with a soil friction angle of 35 degrees and moist unit weight of 110 lbf/ft^3, was subjected to a groundline shear V_G of 500 lbf and a groundline bending moment M_G of 10,000 in-lbf (M_G / V_G = 20 inches).

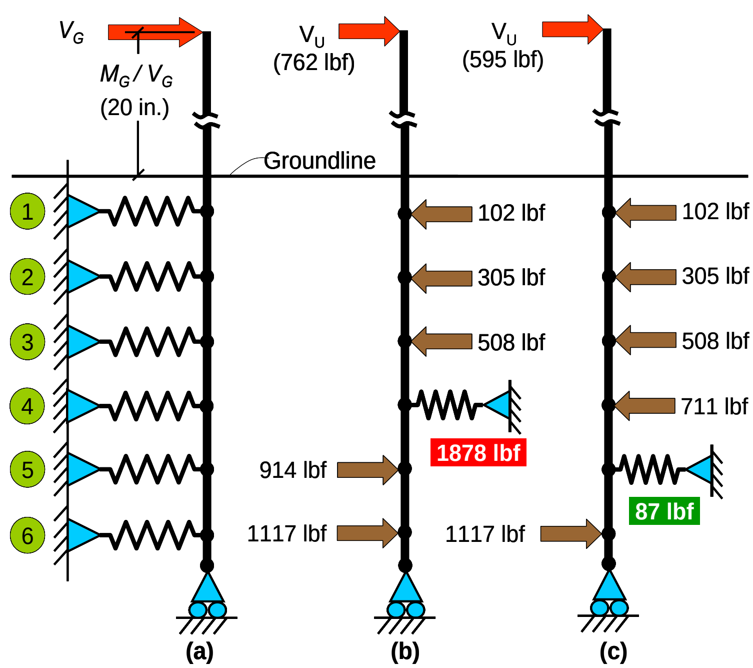


Figure 24. (a) Spring model of nonconstrained post foundation, (b) free body diagram with an overloaded spring 4 as pivot spring, and (c) free body diagram with spring 5 as the pivot spring.

In this case, six springs were used to model the soil as shown in figure 24 (for accuracy purposes, it is recommended that at least 5 springs be used). Table 3 lists the location and ultimate strengths of each spring.

Table 3. Spring Forces in a Nonconstrained Post Foundation [(a)]

Load element	Loca-tion, *z*, inches	F_{ult}, lbf	Pivot Spring		
			4	5	6
			Force in load element, lbf		
Spring 1	4	102	-102	-102	-102
Spring 2	12	305	-305	-305	-305
Spring 3	20	508	-508	-508	-508
Spring 4	28	711	**-1878 [(b)]**	-711	-711
Spring 5	36	914	914	-87	-914
Spring 6	44	1117	1117	1117	**1840 [(b)]**
V_U	-20	NA	762	595	698

[(a)] b = 4.5 inches, d_f = 48 inches, γ = 110 lbf/ft^3, ϕ = 35

[(b)] Force exceeds maximum allowable value.

For the trial-and-error analysis, spring 4 was first selected as the pivot spring. This resulted in a V_U value of 762 lbf and a pivot spring force of -1878 lbf as shown in figure 24. Because the absolute value of the -1878 lbf force exceeds the F_{ult} for spring 4 of 711 lbf (Table 3), spring 4 is not the pivot spring. Consequently, spring 5 was selected as the pivot spring. This resulted in a V_U value of 595 lbf and a pivot spring force of -87 lbf as shown in Figure 24. Because the absolute value of -87 lbf does not exceed the F_{ult} for spring 5 of 914 lbf, spring 5 is indeed the pivot spring. For demonstration purposes, spring 6 was also selected as the pivot spring. The results of this analysis are given in the last column of Table 3.

Multiplication of the V_U value of 595 lbf by the M_G/V_G ratio of 20 inches yields an M_U of 11,900 in-lbf.

Determining M_U and V_U for a Negative M_G/V_G Value

One variation on the preceding “pivot spring” procedure occurs when M_G and V_G independently rotate the foundation in opposite directions, as shown in Figure 25. This produces a negative M_G/V_G ratio. A negative value means that V_G is placed a distance M_G/V_G below the groundline as shown in Figure 25. The rest of the analysis is conducted in the same manner, as if V_G was located a distance M_G/V_G above the groundline.

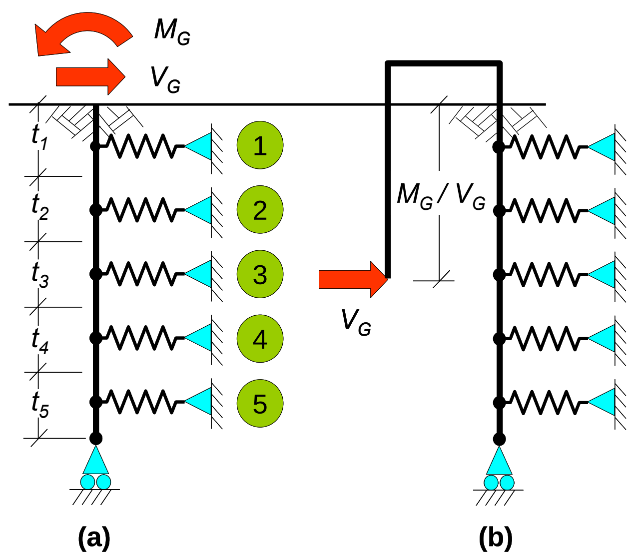


Figure 25. (a) Forces V_G and M_G independently rotate the top of the foundation in opposite directions, and (b) a statically equivalent spring model is used for determination of V_U and M_U.

Example Analysis

Appendices G and H contain examples of soli springs (the Universal Method) being used to determine the lateral capacity and hence adequacy of a foundation.

Increasing Lateral Strength

Generally, the most cost effective way to increase the lateral strength of a post or pier foundation is to increase its effective depth. Two circumstances where increasing depth may not be cost effective are (1) where hole drilling is difficult because of large rock and/or rock strata, and (2) where an increase in depth will require a post that is measurably more expensive because of the increased overall length requirement.

An alternative to increasing foundation depth is to increase foundation width. This can be accomplished with concrete or CLSM backfill, a concrete collar, an uplift anchorage system, or by laminating dimension lumber to the sides of the embedded portion of the post. With respect to the latter, it is important to note that a single 8-foot piece of dimension lumber, when cut in half and appropriately fastened to both sides of a post, effectively increases the foundation width three full inches.

Attaching a post/pier to the footing upon which it bears will effectively increase the depth of a foundation and the foundation width in the footing region. It is important that such an attachment be properly engineered. Friction between a post/pier and footing can not be relied upon for lateral load transfer.

Proposed Changes to ANSI/ASAE EP486.2

Much of the material developed by the author and presented herein has been incorporated into ANSI/ASAE EP486.2. The one major exception is the bulk of the material relating to determination of M_U and V_U via the Universal Method (i.e., via the use of soil springs). This includes the development of a V_U - M_U envelope for a foundation. Because of the value of such an envelope to the overall understanding of a foundation under a combination of loads, procedures for its creation should be included in the commentary of EP486.

For reasons previously articulated, the separate set of resistance and safety factors for lateral strength assessment using the Simplified Method should be removed from EP486.2.

While foundation displacements predicted by the Simplified and Universal Methods will differ for the same foundation, the V_U and M_U produced by the Simplified and Universal Methods will be identical for the same foundation (at least for any foundation that meets the use requirements for the Simplified Method of analysis). This fact should be more clearly articulated in EP486.2.

Summary

A modulus of horizontal subgrade reaction was defined for soil and then used in the development equations for predicting the deformation of shallow post and pier foundations. Because of assumption inherent in their development, use of these equations is limited to certain shallow pier/post foundations.

The modulus of horizontal subgrade reaction was also used to define the stiffness of soil springs used to model the behavior of soil surrounding a foundation. Structural analyses involving soil springs are not limited to specific pier/post foundations.
An ultimate lateral soil resisting pressure was defined for soil surrounding a post/pier foundation and then used to develop equations for calculating the ultimate groundline shear strength V_U and ultimate groundline bending moment M_U for foundations. As with equations developed for predicting foundation movement, the equations for predicting V_U and M_U are limited to certain foundations.

A unique procedure for determining V_U and M_U using a set of soil springs was also developed. The procedure is relatively straight forward, is applicable to any pier/post foundation, and can be completed using a simple calculator.

The concept of a V_U - M_U envelope for a foundation was presented along with techniques for its development. A V_U - M_U envelope facilitates quick assessment of a foundation under a different loadings.

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Appendix A – Derivation of Effective Young's Modulus, E_{SE}

The modulus of horizontal subgrade reaction *k* is defined as the ratio of average contact pressure (between foundation and soil) at depth *z*, p_Z, and the horizontal movement of the foundation at depth z, Δ_Z, or:

$$k = p_Z / \Delta_Z \tag{A-1}$$

The total contact force Q_Z between the foundation and a layer of soil with thickness *t*, is given as:

$$Q_Z = p_Z\, b\, t \tag{A-2}$$

As force Q_Z is disbursed into the soil, soil stress drops as does associated soil straining. At some point, soil deformation is relatively insignificant. Laboratory testing and finite element analyses by many researchers have shown that the vast majority of soil deformation resulting from applied foundation forces will occur within a very short distance of the foundation. For continuous (strip) footings (i.e., situations for which conditions of plane strain apply) there is little deformation below a vertical distance 4*b* of the footing where *b* is the footing width (Schmertmann, et al., 1978). For square and circular footings, this distance reduces to 2*b* where *b* is the diameter/width of the footing. These differences between continuous and square footings are consistent with the differences in stress distributions under continuous and square footings as predicted via elastic theory.

For derivation of an effective Young's modulus, E_{SE}, it is assumed that all soil deformation occurs within a horizontal distance 3*b* of the foundation (see Figure A-1). Terzaghi (1955) states that "the displacements beyond a distance of 3*b* have practically no influence on the local bending moments", and this distance is midway between the aforementioned vertical distances of 4*b* and 2*b* associated with continuous and square footings, respectively. It is important to recognize that the use of a fixed value of 3*b* ignores the reality that the actual horizontal distance of "strain influence" varies. More specially, the horizontal distance of "stain influence" decreases as vertical soil movement is less restrained, which increasingly occurs as you move away from horizontal soil layers characterized by plane strain behavior. Regions of reduced vertical restraint include locations near the ground surface, at the base of the foundation, and at depths where an unrestrained post rotates below grade.

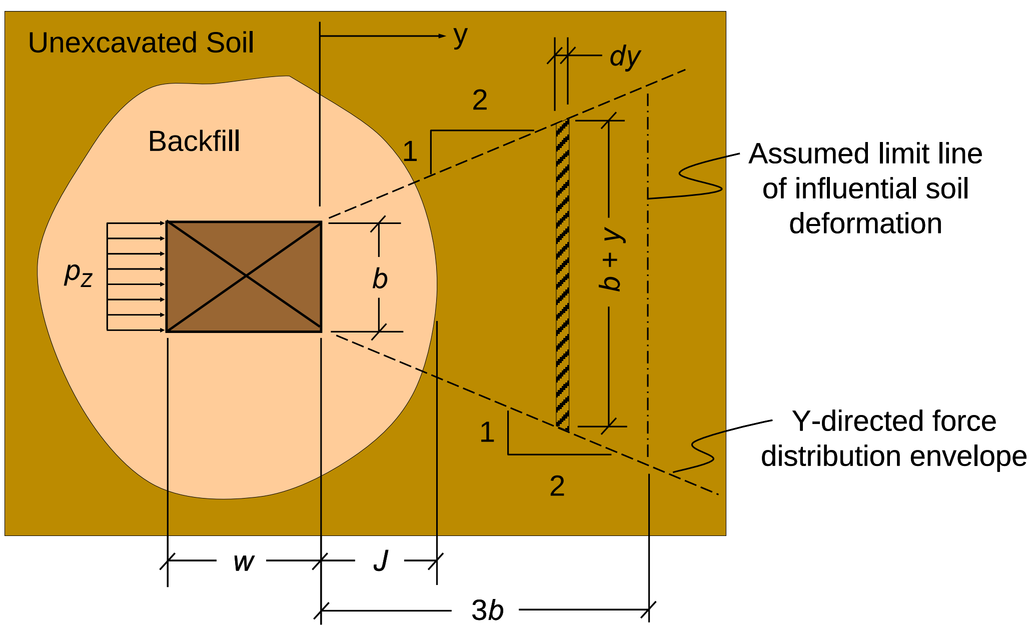

Figure A-1. Differential soil element for determination of effective Young's modulus, E_{SE}.

Figure A-1 shows a differential soil element with width *dy*, length *b* +*y*, and thickness *t*. It is assumed that force Q_Z is uniformly applied along length *b* + *y* of the differential element, which means that that force is uniformly disbursed over an area that increases an amount equal to the distance from the foundation as shown in figure A-1. While this is obviously a crude assumption, it is felt to be sufficiently accurate for the purposes of this derivation, which is to determine how much of the total horizontal movement is due to deformation in the backfill.

Under a uniform load, the differential element is assumed to compress an amount $d\Delta$ given as:

$$d\Delta = \frac{Q_Z \, dy}{C_E \, (b + y) \, t} \tag{A-3}$$

Where C_E is a constant equal to the ratio of uniaxial stress to strain of the differential element.

Integration of equation A-3 yields:

$$\Delta = \frac{Q_Z \ln (b + y)}{C_E \, t} + \text{integration constant} \tag{A-4}$$

If all soil deformation takes place within a distance 3*b* of the foundation (i.e., from y = 0 to y = 3*b*) then total movement Δ_Z of the foundation at depth *z* is given as:

$$\Delta_Z = \frac{Q_Z}{C_E \, t} \left[\ln (4b) - \ln (b) \right] = \frac{Q_Z \ln (4)}{C_E \, t} \tag{A-5}$$

Combining equation 4 (i.e., $k = 2 \, E_S / b$) with equations A-1 and A-2 yields

$$\Delta_Z = \frac{Q_Z}{2.0 \, E_S \, t} \tag{A-6}$$

Equating equations A-5 and A-6 yields the following defining relationship for C_E:

$$C_E = 2.0 \, E_S \ln (4) \tag{A-7}$$

Substituting equation A-7 into A-4 yields:

$$\Delta = \frac{Q_Z \ln (b + y)}{2.0 \, E_S \, t \ln (4)} + \text{integration constant} \tag{A-8}$$

Total foundation movement Δ_Z is the sum of the y-directed deformation in the backfill $\Delta_{Z,B}$ and the y-directed deformation in the unexcavated soil $\Delta_{Z,U}$. Movement $\Delta_{Z,B}$ occurs between the limits of $y = 0$ and $y = J$. Using equation A-8 with E_S equated to $E_{S,B}$ (where $E_{S,B}$ is Young's modulus for the backfill), $\Delta_{Z,B}$ is given as:

$$\Delta_{Z,B} = \frac{Q_Z \left[\ln (b + J) - \ln (b) \right]}{2.0 \, E_{S,B} \, t \ln (4)} \tag{A-9}$$

Movement $\Delta_{Z,U}$ occurs between the limits of $y = J$ and is $y = 3b$ and in similar fashion is given as:

$$\Delta_{Z,U} = \frac{Q_Z \left[\ln (4b) - \ln (b + J) \right]}{2.0 \, E_{S,U} \, t \ln (4)} \tag{A-10}$$

A defining relationship for effective Young's modulus E_{SE} is obtained by substituting E_{SE} for E_S in equation A-6 and rearranging as:

$$E_{SE} = \frac{Q_Z}{2.0 \, t \, \Delta_Z} = \frac{Q_Z}{2.0 \, t \, (\Delta_{Z,B} + \Delta_{Z,U})} \tag{A-11}$$

Substituting equations A-9 and A-10 into A-11 and simplifying yields equation 6a (repeated below)

$$E_{SE} = \frac{1}{I_S / E_{S,B} + (1 - I_S) / E_{S,U}} \tag{6a}$$

where: I_S is referred to as the strain influence factor and is given as $[\ln(1 + J/b)]/\ln(4)$.

If $E_{S,B}$ and $E_{S,U}$ are near equal, then the strain influence factor I_S is a rough approximation of the fraction of total lateral displacement Δ_Z that is due to soil straining within a distance *J* of the face of the foundation. When *J* is equal to 3*b*, the strain influence factor is equal to 1.0, which is consistent with the assumption that all displacement due to soil straining occurs within a distance 3*b* of the foundation. Realize that the actual percentage of total foundation movement that is due to soil straining within a distance *J* of the foundation is dependent on numerous factors including: foundation shape, foundation flexibility, differences in soil elastic properties of the backfill and unexcavated soil, friction between soil and the foundation, magnitude of lateral displacement, foundation restraint conditions, and location relative to both the ground surface and the foundation base.

The strain influence factor is not needed when there is no backfill soil (in which case E_{SE} is equal to E_{SU}) or when the distance from the face of the foundation to the edge of the backfill *J* exceeds 3*b* (in which case E_{SE} is equal to E_{SB}).

Appendix B – Example E_{SE} Calculation

Problem Statement

A post foundation consists of a 3-ply post fabricated from 2- by 10-inch lumber resting on a precast concrete footing. Backfill is 18 inches in diameter and classified as a medium to dense SW-SM soil. The surrounding unexcavated soil is classified as a medium to stiff ML soil. The water table is 2 feet below the footing. What is E_{SE} for the foundation at a location 3 feet below grade?

Effective Young's Modulus, E_{SE}

From Table 1, the backfill material (medium to dense SW-SM soil) has an A_E value of 1320 lbf/in^2/ft (note: the Table 1 value of 660 lbf/in^2/ft is doubled because the soil is located above the water table). Young's modulus at a depth of 3 feet for this backfill, $E_{S,B}$, is equal to the product of A_E and 3 feet or 3960 lbf/in^2. The unexcavated soil (medium to stiff ML soil) has an $E_S = E_{S,U}$ value of 6160 lbf/in^2 that is constant with depth.

With a backfill diameter of 18 inches and post side width of 9.25 inches, the distance J between the edge of the backfill and center of the post face is (18 in. – 9.25 in.)/2 = 4.375 in. Substituting this into equation 7 along with a post face width b of 4.5 inches yields a strain influence factor I_S of 0.49. Substituting this into equation 6a, with an $E_{S,B}$ of 3960 lbf/in^2 and $E_{S,U}$ of 6160 lbf/in^2 yields an effective Young's modulus E_{SE} of 4842 lbf/in^2

Appendix C – Example Structural Analysis - Universal Method

Problem Statement

A nominal 6- by 6-inch No.2 SP post is embedded 4 feet. It rests on a concrete footing but is not attached to the footing. Two nominal 2- by 6-inch wood blocks, 12 inches in length, are bolted to each side of the base of the post to increase the uplift resistance and lateral strength capacity of the foundation. The top 2.5 feet of soil are classified as medium to stiff ML silts. The next several feet of soil below this clay layer are classified as medium to dense SW sands. The water table is located 7 to 8 feet below grade. Backfill is a mixture of the ML silt and SW sand removed by the 18-inch diameter auger used to form the post hole. The mixture is compacted by hand in 6- inch lifts.

If a bending moment of 20,000 in-lbf and a shear force of 1000 lbf are applied to the post foundation at the groundline, what is the resulting rotation and lateral displacement of the post foundation at the groundline?

Spring Location

Two depths are associated with an abrupt change in soil and/or post design properties that will affect spring location: a change in soil type at a depth of 30 inches, and a change in foundation width from 5.5 inches to 12 inches at a depth of 42.5 inches.

The post side width *w* of 5.5 inches equates to a maximum spring spacing of 11 inches (i.e., $t \leq 2w$). To meet this maximum spacing requires a minimum of six springs located as shown in figure B-1.

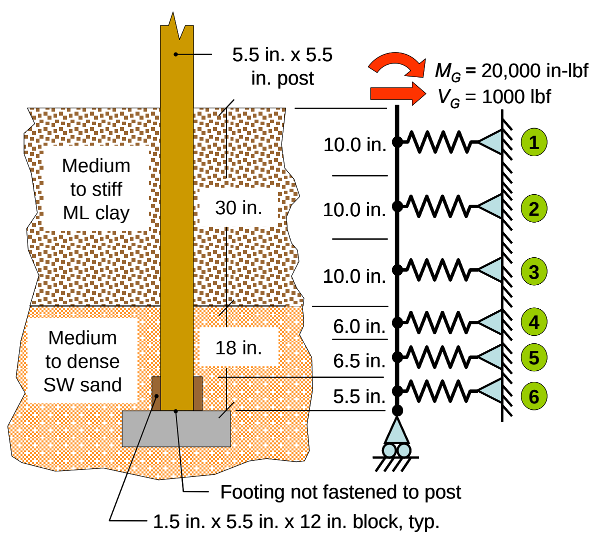

Figure B-1. Non-constrained post foundation and corresponding spring analog.

Presumptive Soil Properties

From Table 1, the medium to stiff ML silt has a Young's modulus of 6160 lbf/in^2, and the medium to dense SW sand has an A_E value of 110 (lbf/in^2)/in. This A_E value is doubled to 220 (lbf/in^2)/inch because the soil represented by the springs is all located above the water table (see Table 1 footnote e).

The mixture of approximately 2.5 feet of ML silt with approximately 2 feet of SW sand is likely to produce a backfill that would grade out as a silty sand (SM). Determination of the exact designation would require knowledge of the particle size distributions of the ML and SW soils prior to mixing. Hand compaction of this backfill in 6-inch lifts should provide a medium to dense consistency. From Table 1, a medium to dense SM soil has an A_E of 55 (lbf/in^2)/in, which is doubled to 110 (lbf/in^2)/inch because the backfill is located entirely above the water table.

E_{SU} and K_H Calculations

Spring number	Thickness of soil layer represented, *t*	Distance from surface, *z*	Width of foundation at spring location, *b*	Increase in Young's Modulus with Depth	
				Unexcavated Soil, $A_{E,U}$	Backfill, $A_{E,B}$
	inches	inches	inches	lbf/in^2/in	lbf/in^2/in.
1	10	5	5.5	-	110
2	10	15	5.5	-	110
3	10	25	5.5	-	110
4	6	33	5.5	220	110
5	6.5	39.25	5.5	220	110
6	5.5	45.25	12	220	110

Spring number	Young's Modulus		Backfill thickness, *J*	Strain Influence Factor, I_s	Effective Young's Modulus, E_{SE}	Horizontal spring stiffness, K_H
	Unexcavated Soil, $E_{S,U}$	Backfill, $E_{S,B}$				
	lbf/in^2	lbf/in^2	inches	-	lbf/in^2	lbf/in
1	6160	550	6.25	0.548	935	18700
2	6160	1650	6.25	0.548	2467	49300
3	6160	2750	6.25	0.548	3669	73400
4	7260	3630	6.25	0.548	4691	56300
5	8635	4318	6.25	0.548	5579	72500
6	9955	4978	4.75	0.241	8024	88267

Structural Analysis

The model shown in Figure B-1 was analyzed using IES Inc.'s VisualAnalysis program (VA, 2013) with the No.2 SP post assigned an *E* value of 1.2 million lbf/in^2. The predicted displaced shape of the foundation post under the 20,000 lbf-in groundline bending moment and 1000 lbf groundline shear force is shown in figure B-2. Groundline displacement and rotation were found to be 0.092 inches and 0.4 degrees, respectively.

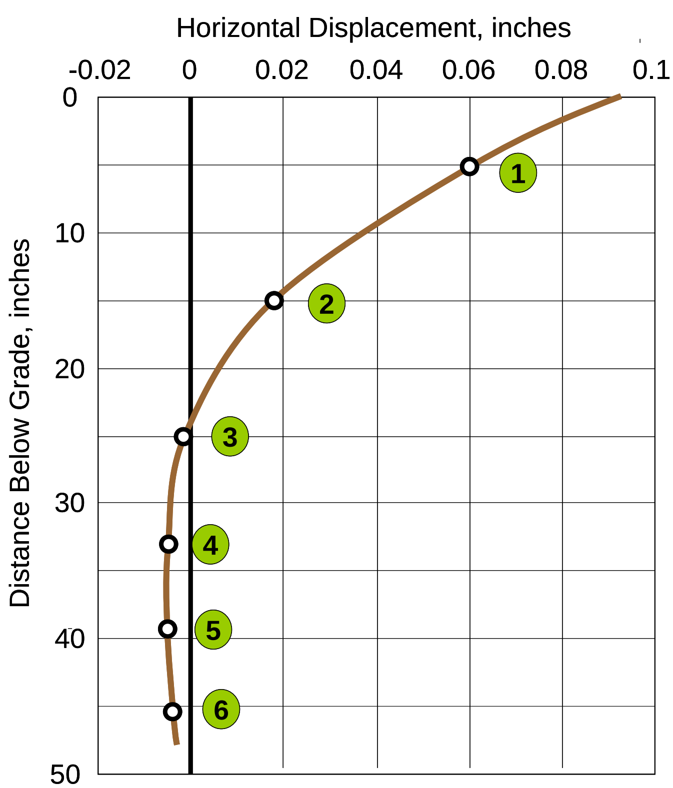

Figure B-2. Displaced shape of the post foundation model shown in figure B-1.

Appendix D – Example Structural Analysis – Simplified Method

Problem Statement 1

A non-constrained nominal 6- by 6-inch solid-sawn post with a modulus of elasticity of 1.2 million lbf/in^2 rests on a precast footing. The surrounding material (soil and backfill) has an effective Young's modulus that increases 155 lbf/in^2 per inch of embedment depth (i.e., A_E = 155 lbf/in^3). What is the maximum depth the post can be embedded in order to use the Simplified Method to approximate post displacements?

Solution

For the Simplified Method equations used to predict foundation movements to be fully applicable, the inequality expressed by equations 1 and 2 must be met. In this case, equation 1 is applicable, that is, embedment depth d must be less than or equal to $2\{E\,I/(2A_E)\}^{0.20}$. Given a moment of inertia I of 76.25 in^4 for a nominal 6- by 6-inch post, the quantity $2\{EI/(2A_E)\}^{0.20}$ is equal to a relatively shallow depth of 24.8 inches.

Problem Statement 2

If the post described in problem 1 is embedded 24 inches (i.e., d = 24 inches), and a groundline bending moment M_G of 20,000 in-lbf and a groundline shear V_G of 1000 lbf are applied to the foundation, what will be the groundline displacement Δ and foundation rotation θ assuming the soil is not overstressed.

Solution

For an A_E of 155 lbf/in^3, equation 17 yields a horizontal displacement Δ of 0.21 inches, and equation 16 a foundation rotation θ of 0.0126 radians (0.72 degrees).

Appendix E – Example Strength Assessment for a Non-Constrained Foundation in Cohesive Soil – Simplified Method

Problem Statement

A non-constrained foundation consists of a nominal 6- by 6-inch post that extends 48 inches below the soil surface and bears on a footing to which it is not attached. Via testing, the surrounding soil (backfill and unexcavated soil) was verified as a medium to dense silt with higher plasticity (soil type MH). If the groundline shear force V_{ASD} and groundline bending moment M_{ASD} due to the applied ASD structural loads are 800 lbf and 45,000 in-lbf, respectively, is the foundation adequate? V_{ASD} and M_{ASD} rotate the foundation in the same direction.

Solution

Since this is an ASD loading, the governing equations are equations 27 and 28 which are given as:

$$M_U \geq f_L\, M_{ASD} \qquad \text{and} \qquad V_U \geq f_L\, V_{ASD}$$

The Simplified Method equations for a constrained foundation in cohesive soils are applicable to this problem because the foundation has a fixed width and the soil is assumed homogenous for the entire depth of the foundation. When the Simplified Method is used, only the first of the preceding governing equations ($M_U \geq f_L\, M_{ASD}$) needs to be checked. This is because the Simplified Method equation for M_U has been derived such that if M_U is greater than $f_L\, M_{ASD}$, then V_U will automatically exceed $f_L\, V_{ASD}$.

When d_{RU} is less than 4b :

$$M_U = b\, S_U\, (4.5\, d^{\,2} - 6\, d_{RU}^{\,2} - d_{RU}^{\,3}/(2b)) \geq 0 \qquad (31)$$

where:

$$d_{RU} = [64\, b^2 + 4V_U/(3S_U) + 12\, b\, d]^{1/2} - 8\, b \leq d$$

When d_{RU} from the preceding equation is greater than 4b:

$$M_U = 9\, b\, S_U\, (d^{\,2}/2 - d_{RU}^{\,2} + 16\, b^{\,2}/9) \geq 0 \qquad (32)$$

where:

$$d_{RU} = V_U/(18\, b\, S_U) + d/2 + 2\, b/3 \leq d$$

In both cases $V_U = f_L\, V_{ASD}$ for ASD

From Table 1 for a medium to dense MH soil, the wet unit weight γ is 105 lbf/ft^3 (0.06076 lbf/in^3) and the undrained soil shear strength S_U is 7 lbf/in^2. From Table 2, f_L = 2.1 which yields a minimum required ultimate groundline soil shear strength V_U of 2.2 x 800 lbf = 1680 lbf. Additionally, b is equal to 5.5 inches and d equals 48 inches. Substituting these variables into the first of the above equations for d_{RU} yields:

$$d_{RU} = [64\, b^2 + 4V_U/(3S_U) + 12\, b\, d]^{1/2} - 8\, b \leq d$$

$$d_{RU} = [64\ (5.5\text{ in})^2 + 4\ (1680\text{ lbf})/(3(7\text{ lbf/in}^2)) + 12(5.5\text{ in})(48\text{ in.})]^{1/2} – 8(5.5\text{ in.}) = 29.65\text{ in.}$$

Since this is greater than 4b = 22 inches, d_{RU} must be recalculated as:

$$d_{RU} = V_U/(18\, b\, S_U) + d/2 + 2\, b/3 \leq d$$

$$d_{RU} = (1680\text{ lbf})/(18(5.5\text{ inch})(7\text{ lbf/in}^2)) + (48\text{ in.})/2 + 2(5.5\text{ in.})/3 = 30.09\text{ in.}$$

and M_U is given as:

$$M_U = 9\, b\, S_U\, (d^{\,2}/2 - d_{RU}^{\,2} + 16\, b^{\,2}/9) \geq 0$$

$$M_U = 9\ (5.5\text{ in.})(7\text{ lbf/in}^2)\ [\ (48\text{ in.})^2/2 – (30.09\text{ in.})^2 + 16\ (5.5)^2/9] = 104{,}100\text{ in-lbf}$$

$$M_U \geq f_L\, M_{ASD} = 2.1\ (45{,}000\text{ in-lbf}) = 94{,}500\text{ in-lbf}$$

Since M_U exceeds 94,500 in-lbf, the foundation is adequate.

Appendix F – Example Strength Assessment for a Constrained Foundation in Cohesionless Soil – Simplified Method

Problem Statement

A surface-constrained foundation consists of a nominal 6- by 6-inch post that extends 48 inches below the soil surface and bears on a footing to which it is not attached. Via testing, the surrounding soil (backfill and unexcavated soil) was identified as a dense, poorly-graded sand (soil type SP). If the groundline bending moment M_{ASD} due to the applied ASD structural loads is 50,000 in-lbf, is the foundation adequate?

Solution

Since this is constrained foundation with ASD loading, the only governing equation is:

$$M_U \geq f_L\, M_{ASD} \qquad (28)$$

Equations for a constrained foundation in cohesionless soils are applicable to this problem because the foundation has a fixed width and the soil is assumed homogenous for the entire depth of the foundation.

$$M_U = d^3\, b\, K_P\, \gamma \qquad (34)$$

where:

$$K_P = (1 + \sin \phi)/(1 - \sin \phi)$$

From Table 1 for a dense, poorly-graded sand, the wet unit weight γ is 120 lbf/ft^3 (0.06944 lbf/in^3) and the drained soil friction angle ϕ' is 35º. From Table 2, f_L = 1.4/(0.82 - 0.01·ϕ) = 2.98. With b equal to 5.5 inches and d equal 48 inches:

$$K_P = (1 + \sin \phi)/(1 - \sin \phi) = 3.69$$

$$M_U = d^3\, b\, K_P\, \gamma = (48 \text{ in.})^3 (5.5 \text{ in.})(3.69)(0.06944 \text{ lbf/in}^3) = 155{,}860 \text{ in-lbf}$$

$$M_U \geq f_L\, M_{ASD} = 2.98(50{,}000 \text{ in-lbf}) = 149{,}000 \text{ in-lbf}$$

Since M_U exceeds 149,000 in-lbf, the foundation is adequate.

Notes:

1. The 2.98 safety factor is a relatively high value, and some engineers feel comfortable using a reduced value in this application. In accordance with the footnote in Table 2, a 20% reduction in f_L is allowed for buildings that represent a low risk to human life in the event of a failure such as a ANSI/ASCE Category I building.
2. Using a bottom collar is an effective way to increase M_U of a constrained foundation, and is common where there is a desire to reduce embedment depth.

Appendix G – Example Strength Assessment for a Constrained Foundation in Cohesionless Soil – Universal Method

Problem Statement

To reduce the embedment depth associated with the Appendix F problem from 48 inches to 36 inches, a reduction in the factor of safety to 2.50 is being applied and a cast-in-place concrete collar that fills the 18-inch diameter of the post hole surrounding the foundation is being added. How far above the footing must this concrete collar extend to provide the necessary ultimate groundline bending capacity $M_U \geq f_L\, M_{ASD}$ = 2.5 (50,000 in-lbf) = 125,000 in-lbf?

Solution

Because the collar results in a foundation with a varying thickness, the Universal Method with its soil springs must be used. With an embedment depth of 36 inches, 6 equally-spaced springs are selected to model the soil, each with a ultimate strength given as:

$$F_{ult} = p_{U,z}\, t\, b \tag{14}$$

where:

$$p_{U,z} = 3\, K_P\, \sigma'_{v,z} \tag{10}$$

$= 3(3.69)\ (0.06944\ \text{lbf/in}^3)\ z = (0.769\ \text{lbf/in}^3)\ z$

K_P = $(1 + \sin\phi)/(1 - \sin\phi) = 3.69$ for $\phi' = 35°$

$\sigma'_{v,z}$ = $\gamma\, z$ for a homogenous soil located above the water table

= $(0.06944\ \text{lbf/in}^3)\ z$

t = Thickness of the soil layer represented by the soil spring

b = Face width of post/pier, footing, or collar that is being modeled with the spring

z = Distance of spring below grade

For the initial check, the collar will be assumed to extend 6 inches above the footing, thus providing the follow spring properties and moment resisting values.

Spring No.	z	t	b	$p_{U,z}$	F_{ult}	$F_{ult}\cdot z$
	in.	in	in.	lbf/in^2	lbf	in-lbf
1	3	6	5.5	2.3	76	228
2	9	6	5.5	6.9	228	2056
3	15	6	5.5	11.5	381	5710
4	21	6	5.5	16.1	533	11191
5	27	6	5.5	20.8	685	18500
6	33	6	18	25.4	2741	90444
					M_U =	128128

The far right column of the above table contains the resisting moment about the groundline provided by each spring. In accordance with equation 39, the summation of these values provides the total ultimate groundline bending capacity M_U of the foundation. Since this exceeds the required value by 3000 in-lbf, the analysis was rerun with a 5.5 inch thick collar. This was quickly accomplished with changes in the depth and associated thickness of the lower two springs.

Spring No.	z	t	b	$p_{U,z}$	F_{ult}	$F_{ult}\cdot z$
	in.	in	in.	lbf/in^2	lbf	in-lbf
1	3	6	5.5	2.3	76	228
2	9	6	5.5	6.9	228	2056
3	15	6	5.5	11.5	381	5710
4	21	6	5.5	16.1	533	11191
5	27.25	6.5	5.5	21.5	769	20945
6	33.25	5.5	18	26.1	2584	85915
					M_U =	126045

The resulting M_U value still exceeds the required minimum of 125,000 in-lbf. However, reducing the collar thickness another half inch does not work as M_U for the foundation with a 5 inch thick collar is 120,600 in-lbfs.

Appendix H – Example Strength Assessment for a Non-Constrained Foundation in Cohesive Soil – Universal Method

Problem Statement

A non-constrained foundation consisting of a 4.5- by 9.25-inch post is subjected to a groundline shear force V_{LRFD} of 1200 lbf, and a groundline bending moment M_{LRFD} of 80,000 in-lbf. How deep must the post extend into the ground if it is attached to an 8 inch thick and 16 inch diameter footing? Via testing, the surrounding soil (backfill and unexcavated soil) was verified as a medium to dense silt with higher plasticity (soil type MH). V_{LRFD} and M_{LRFD} rotate the foundation in the same direction.

Solution

Since this is an LRFD loading, the governing equations for this problem are equations 29 and 30:

$$V_U R_L \geq V_{LRFD} \quad \text{and} \quad M_U R_L \geq M_{LRFD}$$

From Table 2, the LRFD resistance factor for lateral strength assessment R_L is given as 0.68. Thus:

$$M_U \geq M_{LRFD} / R_L = 80{,}000 \text{ in-lbf}/0.68 = 117{,}600 \text{ in-lbf}$$

and

$$V_U \geq V_{LRFD} / R_L = 1200 \text{ lbf}/0.68 = 1768 \text{ lbf}$$

Because the footing results in a foundation with a varying thickness, the Universal Method with its soil springs must be used.

Equations ifor the Simplified Method were used to calculate a post embedment depth assuming the foundation was not attached to the footing. For an undrained soil shear strength S_U of 7 lbf/in^2 (from Table 1 for a medium to dense cohesive soil), the calculated embedment depth was 55 inches. Based on this value, an overall depth (including the attached footing) of 48 inches was selected. The soil spring model used is shown in figure H-1.

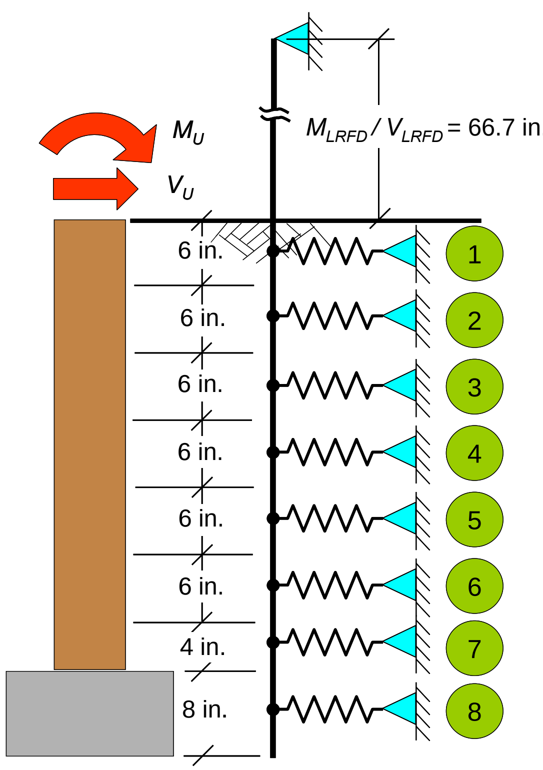

Figure H-1. Spring placement for non-constrained post with attached footing.

Spring strength was calculated in accordance with equation 14 as:

$F_{ult} = p_{U,z}\, t\, b$

where:

$p_{U,z}$ = $3\, S_U\, (1 + z/(2b))$ for $0 \leq z < 4b$
$p_{U,z}$ = $9\, S_U$ for $z \geq 4b$
S_U = 7 lbf/in^2 for a medium to dense cohesive soil (from Table 1)
t = Thickness of the soil layer represented by the soil spring
b = 4.5 inches for post
= 16 inches for footing
z = Distance of spring below grade

Spring No.	z in.	t in.	b in.	$p_{U,z}$ lbf/in^2	F_{ult} lbf
1	3	6	4.5	28	756
2	9	6	4.5	42	1134
3	15	6	4.5	56	1512
4	21	6	4.5	63	1701
5	27	6	4.5	63	1701
6	33	6	4.5	63	1701
7	38	4	4.5	63	1134
8	44	8	16	63	8064

Following calculation of ultimate spring strengths, the free body diagram shown in figure H-2(a) was established. V_U was located a distance M_{LRFD}/V_{LRFD} = 66.67 inches above the groundline. Spring 7 was arbitrarily selected as the pivot spring, and all other springs were replaced with a force equal to their ultimate strength.

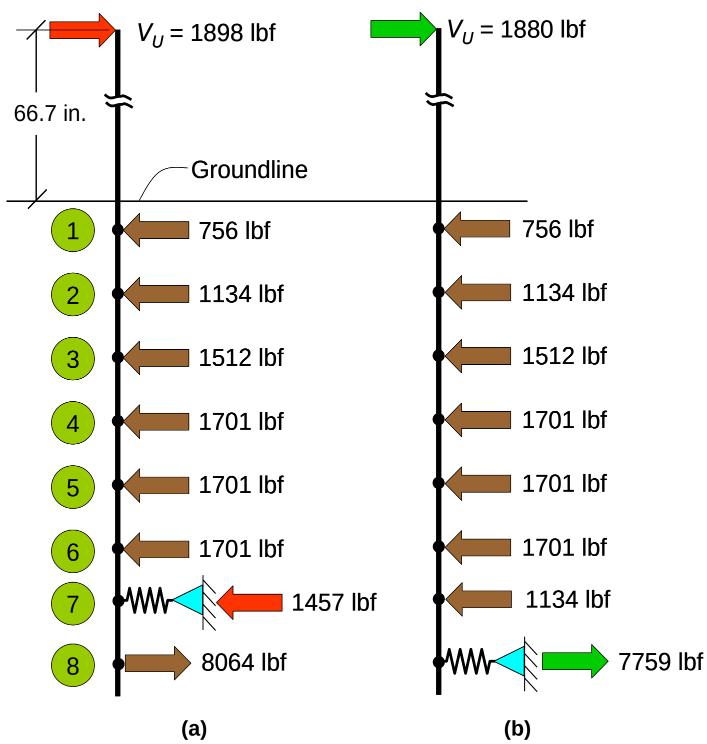

Figure H-2. Spring placement for non-constrained post with attached footing.

Summing moments about the pivot spring in figure H-2(a) results in a V_U value of 1898 lbf. With V_U known, a summation of horizontal forces yields a pivot spring (spring 7) force of 1457 lbf. Since this exceeds the maximum force of 1134 lbf allowed in spring 7, the ultimate pivot point is not located in the soil layer represented by spring 7. Based on the direction at which the spring force of 1457 lbf acts, it is apparent that additional force acting to the left is needed. This is only possible if the ultimate pivot point is lower than the soil layer represented with spring 7. Thus, the ultimate pivot point is located in the layer modeled with spring 8.

A subsequent analysis with spring 8 as the pivot spring was conducted (figure H-2(b)), resulting in a V_U value of 1880 lbf, and a spring 8 force of 7759 lbf. Since the 7759 lbf does not exceed the maximum force of 8064 lbf allowed in spring 8, the ultimate pivot point is indeed located in the soil layer represented by spring 8.

The V_U value of 1880 lbf exceeds the required value of 1768 lbf so the selected overall foundation depth of 48 inches is adequate. A subsequent analysis with an overall foundation depth of 47 inches (post embedment depth of 39 inches) produced a V_U value of 1800 lbf, thereby validating 47 inches as an adequate depth.